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THE

FIELD PRACTICE

OF

LAYING OUT CIRCULAR CURVES

FOR

RAILROADS.

BY

JOHN C. TRAUTWINE, C. E.,

AUTHOR OF "THE CIVIL ENGINEER'S POCKET-BOOK," "A METHOD OF CALCULATING THE CUBIC CONTENTS OF EXCAVATIONS AND EMBANKMENTS," ETC.

THIRTEENTH EDITION.

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PREFACE

TO THE

ELEVENTH EDITION.

THE publishers having informed me that they were about to issue a new edition, I endeavored to dissuade them from it, on the plea that the more comprehensive works of Henck, Searles, and Shunk (all of which, in addition to curves, treat on levelling and other field operations) were better adapted to the purposes of young assistants.

Their reply was that the continued demand for my book proved that some persons preferred to have the subject of curves in a portable form, by itself. Therefore, partly on that ground, and partly from a wish to show how some of the more useful problems may be applied to curves exceeding 180°, I assented to a new edition, and, rather hastily, prepared this.

The extension beyond 180° has not, I believe, been hitherto attempted, although its utility has of late years been made evident in the tortuous canyons of our Western States and of Mexico.

The additional matter has nearly doubled the number of pages.

The number of problems might be indefinitely increased by the aid of Euclid, or of any good modern work on geometry; but in fact very few are required in actual practice. Any extraordinary ones that may present themselves can be solved by a drawing. In preparing his drawing for this purpose, the young assistant need not always confine himself to such scales as may be managed by the common dividers; but when, as often happens, only a few chains of the curve need be drawn (including turnouts, etc.), he may with great ease lay them off on the same principle as in field operations, by using his

In Hutton's Tables, Fifth Edition, 1811.

Sine of 6° 8′, for ·1063425, read ·1068425.
Page 328, at top, for 25 Deg., read 40 Deg.
Tangent of 44° 60′, for ·1000000, read 1·000000.
Tangent of 41° 60′, for ·8994040, read ·9004040.

In Dr. Gregory's Corrected Edition (the 8th) of Hutton's Tables, 1838.

Sine of 49° 14′, for ·7576751, read ·7573751.

In Hassler's Tables, 1830.

Sine of 78° 24′, read ·9795752.
Sine of 20° 60′, " ·3583679.
Sine of 66° 19′, " ·9157795.
Sine of 56° 39′, " ·8353279.
Sine of 55° 20′, " ·8224751.
Sine of 53° 4′, " ·7993352.
Sine of 48° 12′, " ·7454760.
Sine of 45° 3′, " ·7077236.

It is scarcely necessary to remark that, beyond 44°, the Sines, Tangents, etc., are read *upwards*, from the bottom of the page, using the corresponding column of minutes. To find the sine of an angle exceeding 90°, subtract the angle from 180°, and take out the sine of the remainder—because the sine of an angle, and that of what it wants of 180°, are the same. JOHN C. TRAUTWINE.

PHILADELPHIA, 1851.

REMARKS.

The principle upon which railroad curves are laid out, is found in Euclid. It was employed in 1761, in tracing the northern boundary of the State of Delaware. Col. Stephen H. Long, of the U. S. Army, was the first person who reduced it, by means of appropriate rules and tables, to the form now in general use. Professor Rankine, in his "Civil Engineering," claims to have been the first to publish the method in 1843; but states that he had used it in 1841. Col. Long's "Railroad Manual," with full rules and tables for curves, was published early in 1829; and was in general use among Engineers throughout the United States for twelve years before the earliest date claimed by Prof. Rankine. Samuel W. Mifflin, C. E., of Pennsylvania, also published his "Railway Curves," based on the same principle, in 1837.

My first edition was in 1851. Mr. Henck's widely known standard "Field-Book for Railroad Engineers" followed in 1854.

J. C. T.

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FIELD PRACTICE

OF

LAYING OUT CIRCULAR CURVES

FOR

RAILROADS.

xi

LAYING OUT CIRCULAR CURVES
FOR
RAILROADS.

CHAPTER I.

PRINCIPLES OF LAYING OUT CURVES.

ARTICLE I.

METHOD 1.

To lay out a Curve by means of Tangential Angles.

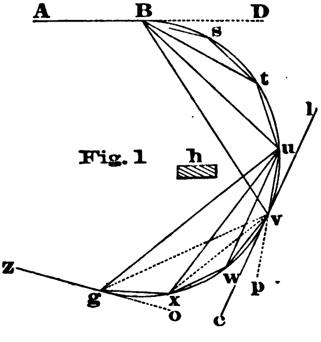


Fig. 1

IF from any point B, Fig. 1, in a straight line A D, we lay off any number of equal angles, as D B *s*, *s* B *t*, *t* B *u*, *u* B *v*, etc., and at the same time make the chords B *s*, *s t*, *t u*, *u v*, etc., equal to each other; then the points B, *s*, *t*, *u*, *v*, etc., will be situated in the circumference of a circle, which is tangential to the line A D at the point B.

The first of these angles, D B *s*, is called the *tangential angle*, as being that by which the curve is connected with the tangent A D; but inasmuch as the others are all equal to it, they also are called tangential angles.

If any obstacle, as *h*, should prevent our seeing from B farther than to *v*, the curve may be continued by removing the instrument to *u*, the point preceding *v*; thence sighting first on *v*, continue to lay off additional tangential angles *v u w*, *w u x*, etc., as before. Or else, moving the instrument to *v* itself instead of to *u*, sight back to *u*, and lay off first the exterior angle *p v w*, equal to *double* the tangential angle, and afterward continue the tangential angles *w v x*, *x v g*, etc., as before, to the end of the curve.

Finally, in order to pass from the end of the curve at *g*, on to a tangent *g z*, place the instrument at *g*, and sighting back to *x*, lay off the tangential angle *x g o*; then *o g* continued toward *z* will be the required tangent. (See Art. IV.)

For the tangential angles corresponding to different radii, and chords of 100 feet (the length adopted in this book), see page 18.

Proof of Method 1.—Equal angles, *s* B *t*, *t* B *u*, etc., at the circumference of a circle, are subtended by equal chords, *s t*, *t u*, etc. *Euclid.*

Remark.—In practice it will be more accurate to remove the instrument to *v*; sight back to B, and lay off the angle B *v l* equal to D B *v*, thus bringing the telescope to sight along *v l*. Then *v l* will be a tangent to the curve at *v*. Revolve the telescope, and it will then sight along *v c*, which is a continuation of the tangent *v l*. Then from *v* lay off tangential angles *c v w*, *w v x*, *x v g*, etc., as before; at the same time making the chords *v w*, *w x*, *x g*, each 100 feet.

ARTICLE II.

METHOD 2.

To lay out a Curve by means of Deflection Angles.

Fig. 2. First, having, as in Method 1, laid off a tangential angle D B *s*, and measured the chord B *s*, remove the instrument to the end *s* of the chord, and make the exterior angle *m s t* equal to *twice* the tangential angle, and measure the chord *s t*; and so on at the other points *t*, *u*, *v*, etc., making each of the exterior angles *n t u*, *o u v*, etc., equal to twice the tangential angle, and all the chords equal; then will the points B, *s*, *t*, *u*, *v*, etc., be in the circumference of a circle which is tangential to the line A D at the point B, as by the first method.

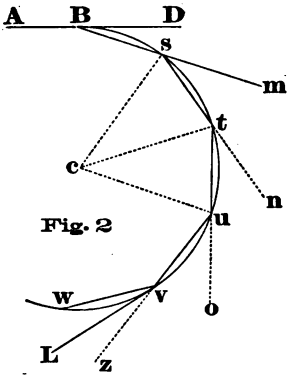


Fig. 2

But if, at any of these points, as v, we wish to pass off to a tangent v L, employ at that point the *tangential* angle $z\,v$ L, equal to half the deflection angle $z\,v\,w$. (See Art. IV.)

These exterior angles, included between any *chord* and the extension of the preceding *chord*, are called *chord deflection angles*, or simply *deflection angles*, or sometimes *angles of curvature*. In any given circle, the angle of deflection is always precisely double the tangential angle, supposing the chords to be equal. At page 18, we give tables of the angles corresponding to circles of different radii, embracing the limits of railroad practice; and calculated for chords 100 feet in length, that being the usual length for a measuring-chain on public works.

Proof of Method 2.—Equal angles, $t\,c\,u$, $t\,c\,s$, etc., at the center of a circle, as well as those at the circumference, are subtended by equal chords, $t\,u$, $t\,s$, etc.; and the deflection angles, $n\,t\,u$, $m\,s\,t$, etc., are equal to the angles, $t\,c\,u$, $t\,c\,s$, etc., at the center of the circle, subtended by one of the equal chords $t\,u$ or $t\,s$. This angle at the center, so subtended, is called the *chord central* angle. The tangential angle, being always half the deflection angle, is, of course, always half this central angle. *The deflection angle gives the curve its name;* thus a 3°, 4°, or 10° curve is one whose *deflection* angle is 3°, 4°, or 10°.

ARTICLE III.

METHOD 3.

To lay out a Curve without a Transit.

The *deflection* angles, Fig. 3, *e s t*, *f t u*, *g u v*, *h v w*, etc., being double the *tangential* angle D B *s*, the *arcs e d t*, *f i u*, *g m v*, *h n w*, etc., are double the *arc* D *c s*, since the arcs of circles are proportionate to the angles which they subtend; but the *chords e t*, *f u*, *g v*, *h w*, etc., are *not* double the *chord* D *s*, since the chords of arcs are not proportionate to the arcs, or to the angles which they subtend.

The chords *e t*, *f u*, *g v*, *h w*, etc., which subtend the deflection angles, are called *deflection distances;* and the chord D *s*, which subtends the tangential angle, is called the *tangential distance.*

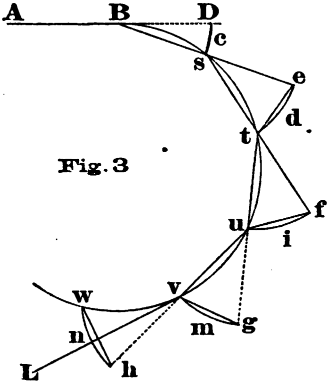


Fig. 3

But although, in any given circle, the deflection distance is not *truly* twice the tangential distance, yet the difference is so trifling in large railroad curves, with chords of but 100 feet, that it may be entirely neglected in curves of more than 300 feet radius, as seen in the table, page 18. In that table the correct length of both will be found for different radii, and for chords of 100 feet.

Having these respective distances, we may frequently trace a curve on the ground by the eye only, with very tolerable accuracy, sufficient for guiding the excavations and embankments, especially on nearly level ground. Suppose, for instance, it be required to lay out in this manner a curve of 5730 feet radius.

First, find by the table, page 18, or by Art. XIII., the deflection distance *e t*, or *f u*, etc., corresponding to a radius of 5730 feet for a chord of 100 feet, viz., 1·745 feet; and also the tangential distance D *s* ·872 of a foot.

Then from the starting point B, and in line with A B, measure B D equal 100 feet; and put a chain-pin at D. Also from B, measure the chord B *s*, equal 100 feet; at the same time measuring with a graduated rod, from the pin D, the *tangential* distance D *s*, equal to ·872 of a foot; and place a stake at *s*. The pin at D may then be removed.

Next, make *s e* equal 100 feet, placing a pin at *e*, precisely in line with *s* B; also from *s* measure *s t* equal 100 feet; at the same time measuring with the rod from the pin *e*, the *deflection* distance *e t*, equal to 1·745 feet. Place a stake at *t*, and remove the pin at *e*. In this manner proceed to find other points as far as the end of the curve at *v*.

In order to pass from the curve, as at *v*, to a tangent *v* L, proceed as before, only using the *tangential* distance *h n*, instead of the deflection distance *h w*. (See Art. IV.)

This method is abundantly accurate for laying out curves on a canal, or common road; and will occasionally answer very well, when carefully performed, for railroad curves, in the absence of an instrument. Thin straight rods, iron-pointed, and a plumb line should be used for ranging the points in the latter case.

Rules for calculating radii, distances, and angles, are given further on.

TABLE OF RADII, Etc.—Chord 100 Feet.

(CONTINUED.)

The Tangential Angle is always one-half of the Angle of Deflection.

Angle of Deflection.	Radius in feet.	Deflection distance in feet.	Tangential distance in feet.	Angle of Deflection.	Radius in feet.	Deflection distance in feet.	Tangential distance in feet.
° ′				° ′			
5 55	968·8	10·32	5·163	12 30	459·3	21·77	10·90
6	955·4	10·47	5·235	45	450·3	22·21	11·12
5	942·3	10·62	5·308	13	441·7	22·64	11·34
10	929·6	10·76	5·380	15	433·4	23·07	11·56
15	917·2	10·90	5·453	30	425·4	23·51	11·77
20	905·1	11·04	5·526	45	417·7	23·94	11·99
25	893·4	11·20	5·600	14	410·3	24·37	12·21
30	881·9	11·34	5·672	15	403·1	24·81	12·43
35	870·8	11·48	5·744	30	396·2	25·24	12·65
40	859·9	11·63	5·817	45	389·5	25·67	12·86
45	849·3	11·78	5·890	15	383·1	26·11	13·08
50	839·0	11·92	5·962	15	376·8	26·54	13·30
55	828·9	12·06	6·035	30	370·8	26·97	13·52
7	819·0	12·21	6·108	45	364·9	27·40	13·73
5	809·4	12·36	6·180	16	359·3	27·83	13·95
10	800·0	12·50	6·253	30	348·5	28·70	14·38
15	790·8	12·64	6·326	17	338·3	29·56	14·82
20	781·8	12·79	6·398	30	328·7	30·43	15·25
25	773·1	12·94	6·470	18	319·6	31·29	15·69
30	764·5	13·08	6·544	30	311·1	32·15	16·12
35	756·1	13·22	6·616	19	302·9	33·01	16·56
40	747·9	13·37	6·689	30	295·3	33·87	16·99
45	739·9	13·51	6·762	20	287·9	34·73	17·43
50	732·0	13·66	6·835	21	274·4	36·44	18·30
55	724·3	13·80	6·907	22	262·0	38·17	19·17
8	716·8	13·95	6·980	23	250·8	39·87	20·04
15	695·1	14·38	7·198	24	240·5	41·58	20·91
30	674·7	14·81	7·416	25	231·0	43·28	21·77
45	655·4	15·25	7·634	26	222·3	44·98	22·64
9	637·3	15·68	7·852	27	214·2	46·68	23·51
15	620·1	16·12	8·070	28	206·7	48·38	24·37
30	603·8	16·55	8·288	29	199·7	50·07	25·24
45	588·4	16·99	8·506	30	193·2	51·76	26·11
10	573·7	17·43	8·724	31	187·1	53·45	26·97
15	559·7	17·87	8·942	32	181·4	55·13	27·83
30	546·4	18·30	9·160	33	176·0	56·82	28·70
45	533·8	18·73	9·378	34	171·0	58·47	29·56
11	521·7	19·17	9·596	35	166·3	60·14	30·42
15	510·1	19·61	9·814	36	161·8	61·80	31·29
30	499·1	20·05	10·03	37	157·6	63·46	32·15
45	488·5	20·47	10·25	38	153·6	65·11	33·01
12	478·3	20·91	10·47	39	149·8	66·76	33·87
15	468·6	21·34	10·69	40	146·2	68·40	34·73

ARTICLE IV.

On Sub-Chords.

We have hitherto spoken of curves as if they were composed of equal chords, each 100 feet in length. It frequently happens, however, that at the end of a curve, as at *e*, Fig. 4, we are obliged to use a shorter, or sub-chord, *d e*, in order to unite properly with the tangent *e f*.

In that case, and *when using Method* 1, *Art. I., of laying off curves by means of tangential angles*, we must, in order to fix the point *e*, lay off at A (where the instrument stands), a *sub-tangential* angle *d* A *e*, as much smaller than the entire tangential angle B A *c*, or *c* A *d*, etc., as the sub-chord *d e* is smaller than an entire 100 feet chord, A *c*, *c d*, etc. Thus if the sub-chord be one-half, or one-fourth, etc., of the entire chord, the sub-tangential angle must be one-half, or one-fourth, etc., of the entire tangential angle.

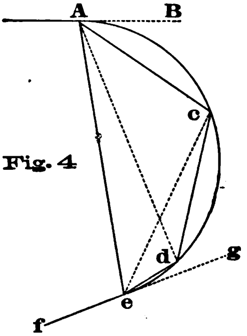


Fig. 4

This method is not mathematically exact, for the reason stated in Art. III. (viz., that the *chords* subtending different angles are not proportional to those angles); yet, for curves of 300 or more feet radius, and with chords not exceeding 100 feet in length, the error may be overlooked in practice. Should, however, greater accuracy be required at any time, or for radii less than 300 feet, see Art. V.

In like manner, when we pass off from a sub-chord, as at *e*, to a second tangent, *e f*, we must place the instrument at *e*, and lay off the same sub-tangential angle *d e g;* or, which is better, take sight from *e* to *c*, and lay off the angle *c e g*, equal to the *sum* of a tangential and the sub-tangential angle.

But *when using Method* 2, *Art. II., of deflection angles, or Method* 3, *Art. III., of deflection distances, we may calculate the sub-deflection angle* a s e, *Fig.* 5, *and sub-deflection distance* a e, *formed between a sub-chord* s e, *and the extension*

ARTICLE V.

To find Sub-Tangential Angles exactly.

Rule.—Divide *half* the sub-chord by the radius. The quotient will be the sine of the sub-tangential angle. Therefore, find in the table of sines, etc., page 124, the sub-tangential angle opposite this sine.

Example.—What is the sub-tangential angle for a sub-chord of 70 ft.; radius 146·19 ft.?

Here, half the sub-chord is 35 ft.; hence,

$\frac{35}{146{\cdot}19} = {\cdot}2394$; and opposite ·2394 in the table of sines, etc., we find 13° 51′, the sub-tangential angle required.

To find Sub-Deflection Angles exactly.

A sub-deflection angle is equal to the chord-tangential angle + the sub-tangential angle.

Rule.—Divide *half* the sub-chord by the radius. The quotient will be the sine of the sub-tangential angle. Opposite this sine in the table find the angle itself. Add it to the whole-chord tangential angle.

Example.—What is the sub-deflection angle for a sub-chord of 70 ft.; radius 146·19 ft.; chord tangential angle 20°?

Here, $\frac{35}{146{\cdot}19} = {\cdot}2394 =$ sine of $13° 51' =$ sub-tangential angle.

And $20° + 13° 51' = 33° 51' =$ the sub-deflection angle required.

To find Sub-Tangential Distances exactly.

Rule.—Divide *half* the sub-chord by the radius. The quotient will be the sine of the sub-tangential angle. Find this angle. Find the sine of *half* this angle. Multiply it by 2. Multiply the product by the sub-chord.

Example.—What is the sub-tangential distance for a sub-chord of 70 ft.; radius 146·19 ft.?

Here, $\frac{35}{146{\cdot}19} = {\cdot}2394$, or the sine of 13° 51′. Half of this is 6° 55½′; the sine of which is ·1206. And ·1206 × 2 = ·2412. And ·2412 × 70 = 16·884 ft., the sub-tangential distance required.

To find Sub-Deflection Distances exactly.

Rule.—Divide *half* the sub-chord by the radius. The quotient will be the sine of the sub-tangential angle. Opposite this sine find the angle itself in the table of sines. Add to this angle a whole-chord tangential angle. The sum will be the sub-deflection angle. Find the sine of *half* this angle. Multiply this sine by 2. Multiply the product by the sub-chord.

Example.—What is the sub-deflection distance for a 70 ft. sub-chord; radius 146·19 ft.; whole-chord tangential angle 20°?

Here, $\frac{35}{146{\cdot}19} = {\cdot}2394$ = sine of 13° 51′, sub-tangential angle.

And 13° 51′ + 20° = 33° 51′, sub-deflection angle; half of which = 16° 55½′.

And sine of 16° 55½′ = ·2911. And ·2911 × 2 = ·5822.

And ·5822 × 70 = 40·754 ft., the sub-deflection distance required.

For proofs of the foregoing rules, see next page.

Proofs of the Foregoing Rules.

Let *b a c*, Fig. 5½, be any circular curve; *a b* a 100 ft. chord; *a c* a sub-chord; and *x f*, *t n*, two tangents to the curve at *c* and *a*. Let the whole chord *a b* be extended, as to *s*. Then *n a c*, or *x c a*, or *a b c*, is the sub-tangential angle of the sub-chord *a c; s a n*, equal to *t a b*, is the tangential angle of the whole chord *a b; s a c* is the sub-deflection angle; a **straight** line from *n* to *c* is the sub-tangential distance; and *s c* is the sub-deflection distance.

Of Ist Rule.—Because the angle at the center of the circle, and opposite to half the sub-chord, is equal to the sub-tangential angle. Therefore the sine of the first (which is the one we actually find,) is also the sine of the second.

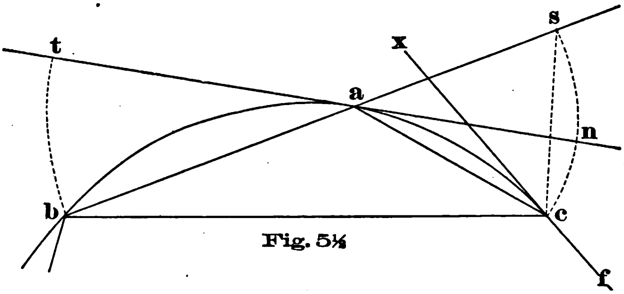


Fig. 5½

In any circle, the *half* of any chord, divided by the radius, gives the natural sine of the angle at the center, and opposite to the half-chord.

Of 2d Rule.—It is self-evident that the sub-deflection angle *s a c* is equal to the chord-tangential angle *s a n* (= *t a b*) + the sub-tangential angle *n a c*.

Of 3d Rule.—The sub-tangential distance is the **chord** *c n* of the sub-tangential angle *n a c;* and, in any circular arc *s n c*, any chord *c n* is equal to *twice* the sine of *half* the subtended angle *n a c* × radius *a c* of the circle.

Of 4th Rule.—The sub-deflection distance *s c* is the chord of the sub-deflection angle *s a c;* and, on the same principle as the foregoing, in the circular arc *s n c*, any chord *s c* is equal to *twice* the sine of *half* the subtended angle *s a c*, × radius *a c* of the circle.

ARTICLE VI.

Ordinates for Entire Chords.

It would be both tedious and liable to inaccuracy to attempt to fix all the necessary points in railroad curves by the foregoing means, which are employed only for entire chords, or for such sub-chords as may be required at the ends of curves.

The best method is to stretch a piece of twine *a b*, Fig. 6, 100 feet long, between two adjacent chord-stakes, and measure off, as nearly as may be at right angles to it, with a graduated rod, the previously calculated ordinates *c d*, *e f*, *g h*, etc., placing pegs at *d*, *f*, *h*, etc. On the tops of these stakes, small tacks are driven to define the precise point in the curve. Our table of ordinates, page 50, is calculated for distances apart, *b c*, *c e*, *e g*, etc., of 5 feet; and for all curves likely to occur in practice. The 5 feet distances on the twine should be marked by knots or otherwise; and those at the center, and half way between it and the ends, be further distinguished by tying on pieces of tape.

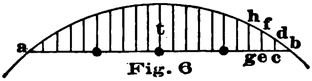


Fig. 6

The 5 feet distances are only used (after the excavations and embankments are finished) for placing pegs to guide the laying of the rails, and then only for very sudden curves; for those of large radii, distances of 10 feet are quite sufficient, or even 25 feet for very easy curves. For guiding the curves of the cuttings and fillings, it is not necessary to place the stakes nearer than 50 feet apart; unless for those of less than about 1000 feet radius, when they may be placed 25 feet apart. Ordinates for other radii, or for angles of deflection, intermediate of those in the table, may either be calculated by the rules given further on, or they may be taken proportionally intermediate of the tabular ones, with sufficient accuracy for practice.

To calculate ordinates for chords and sub-chords, see Articles XV., XVI., XVII.

CHAPTER II.

ARTICLE VII.

HAVING shown the general principle on which curves are run, we will now state a few simple elementary points connected with them, with which the young assistant should make himself *perfectly familiar* before entering upon the problems in Chapter III.

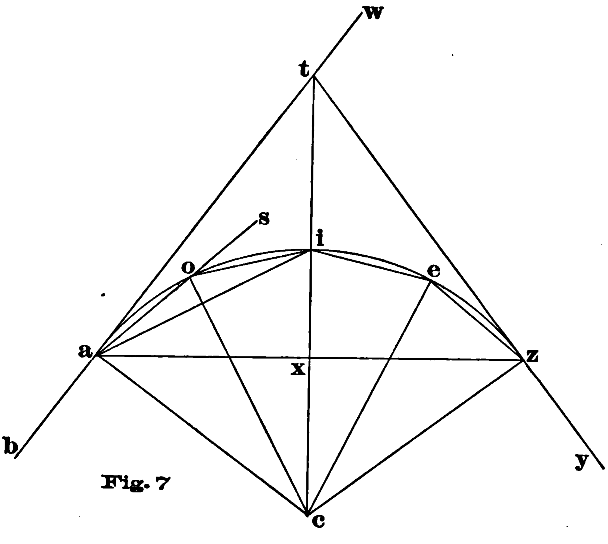


Fig. 7

The length of a curve is supposed to be measured, not along its actual curved line or arc, but along the chords. Thus the curve *a i z*, in Fig. 7, if supposed to have been run with 4 chords of 100 feet each, is said to be 400 feet long, although it is evident that the arc itself is a little longer. But in practice the difference may generally be disregarded; for even in a curve of but 300 feet radius it amounts to only about $5\frac{1}{4}$ inches to a chord of 100 feet; and with 2000 feet radius to but about $\frac{1}{8}$ of an inch.

The length of a curve may also be expressed in degrees of its central angle a c z, contained between two radii *c a, c z,* drawn from the center *c* to the ends *z* and *a* of the curve. Thus if the curve in Fig. 7 be 4 chains of 25° deflection angle, its central angle *a c z* will be 25° × 4 = 100°; and the curve may therefore be said to be 100° long.

The beginning of a curve, or the end first reached in the survey, is called the **Point of Curve,** or *P. C;* and its end is called the **Point of Tangent,** or *P. T.* They are distinguished on the ground by having those letters printed on the stakes with red chalk.

The field operation is called the *tracing, turning, running, laying out,* or *staking out* of curves.

In Fig. 7, let *b t* and *y t* be two tangents touching the ends *a* and *z* of any curve (of *less than* 180° long), and extended to meet at *t;* and let *a c* and *z c* be two radii drawn from the ends of the curve to the center *c* of the circle. Then the angle *a c z,* subtended by the entire curve *a o i e z,* is called the **central angle** *of the curve,* or the *total* or *entire central angle.* This distinguishes it from the small central angle *a c o, o c i, i c e,* or *e c z,* subtended by only one chord *a o* or *o i,* etc., and which is called the *chord central angle,* or *central angle of a chord.* It is generally known at the time, of which of these two angles we are speaking, and therefore the simple term *central angle* is usually applied to either of them.

The angle *a t z,* Figs. 7 and 8, between two tangents, *t a, t z,* is called the **apex angle** of the curve. If only either one of the tangents is prolonged beyond *t,* as *t a* is prolonged to *w,* then the outer angle *w t z,* so formed, may be called the **outer meeting angle** of the tangents, inasmuch as it is the *outer* angle at which the two tangents *meet* (not cut, cross, or intersect each other), while the apex angle *a t z* is the *inner* meeting one.

The Total Deflection Angle, *w t z,* Fig. 7 (in distinction from the *chord* deflection angle, *s o i*), denotes the total number of degrees that the line of survey, or of road, deflects from its previous direction; and *is equal to the total central angle* (*a c z,* Fig. 7, or *n t o,* Fig. 30, p. 69) subtended by the whole curve, *a i z* or *n w o.* When, as in Fig. 7, the word *natural* before sines, etc.; and in referring to any angle, as act, Fig. 8½, we shall omit the word *angle*, and call it simply act, etc.

If any two tangents be drawn to a circle from any one point, as ta, tz, from t, Fig. 8½, they will be of the same length.

To find ti, first find the secant tc, and from it take the radius ci. **Or,** ti is $= ta \times$ nat tangent either of tai, or of *half* taz, or of *quarter* of wtz, or of *quarter* of acz. For a proof of this, see demonstration of Article XXI.

In Fig. 8½, az is the chord of the entire curve aiz, and it is plain that either taz or tza is the tangential angle of said chord, or the angle at which it leaves a tangent ta or tz. Hence taz or tza is equal to half the deflection angle wtz, of the entire curve; or to half its central angle acz, in the same way that the tangential angle tas or sai of a 100 ft. chord ao or oi, is equal to half the chord-deflection angle soi, or to half the chord-central angle aco or oci. Therefore the *sum* of taz and tza is $= wtz$ or acz.

On the same principle, if chords ai, zi, be drawn from the ends a and z to the middle i of the entire curve, then tai, or its equal tzi, will be $= iaz$, or $= iza$; in other words, $=$ *half* the tangential angle taz or tza of the entire curve, or $=$ *one-fourth* of wtz, or of acz; and the *sum* of tai and tzi is $= taz$, or $=$ *half* the central angle acz.

If the chords, as ae and ze, are drawn to a point, as e, *not* in the middle of the curve, the angles tae and tze, between the chords and the tangents, will no longer be equal, but their *sum* will still be $= taz$, or $=$ *half* acz.

It will be observed that tai or tzi is $= aco$, or $=$ half aci, or $=$ half icz; while tae is $=$ half ace; and tze is $=$ half ecz.

The two triangles atx and cax, Fig. 8½, will always be similar; that is, they will have the same angles, and be alike in every respect except size.

Therefore, as $ac : at :: xc : ax$, and $:: ax : xt$;
and, as $ac : xc :: at : ax$;
and, as $ac : ax :: at : xt$.

Also, in a **compound** curve $a\,e\,z$, Fig. $8\frac{3}{4}$, the sum of $t\,a\,z + t\,z\,a$ is $=$ the sum of the *two* central angles $v + w$; or $=$ the total deflection angle x. And if, from the ends of the curve, two chords $a\,e$, $z\,e$, be drawn *to the point* e *of compound curvature*, then is $t\,a\,e + t\,z\,e = \frac{1}{2}$ the sum of the *two* central angles w and v; or $=$ *half* the outer meeting angle x. This will not hold good if $a\,e$ and $z\,e$ be drawn to any other point than that of compound curvature.

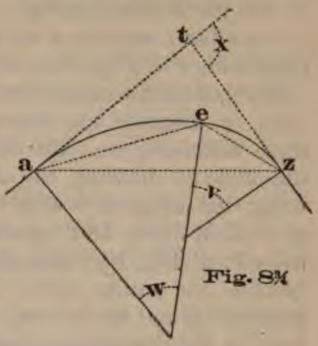


Fig. 8¾

ARTICLE IX.

To find the Total Deflection Angle.

This is usually either given by the field notes, or found from the map, in the office. It is defined on page 29; and, as there stated, it is always equal to the total central angle of the curve, whether this be greater or less than 180°.

It is equal also to the chord-deflection angle of the curve multiplied by the number of 100 ft. chords contained in the curve.

Or, **having the apex distance** $a\,t$, Fig. 8 or $8\frac{1}{2}$, **and the radius** $a\,c$, first find the *outer meeting* angle, thus: Divide the apex distance by the radius. The quotient is the natural tangent of $a\,c\,t$, or of half the outer meeting angle. From the table of tangents take the angle corresponding to this nat tangent. Multiply the angle by 2. If the product is 180°, or less, it is at once the total deflection angle; but if it exceeds 180°, take it from 360°. Then the remainder will be the total deflection angle.

The point t, Fig. $8\frac{1}{2}$, is frequently inaccessible, or imaginary, as shown by the point b, Figs. 9 to 12; in which

Rule 4.—Having $a\,z$ and $x\,i$, Fig. $8\frac{1}{2}$. Then,

$$\text{Radius} = \frac{\text{Square of } \textit{half } a\,z + \text{Square of } x\,i}{\text{Twice } x\,i.}$$

This Rule applies to the radius of any circular arc, of which we know the chord and rise.

Rule 5.—For a track already laid, measure at the rails a chord of 100 feet, and its middle ordinate in feet. Refer to our Table of Ordinates, p. 50, for the deflection angle, opposite which, in table, p. 18, is the radius.

ARTICLE XI.

To find the Apex (or Tangent) Distance of a Curve.

Rule 1.—Multiply the radius by the tangent of *half* the total central angle.

Rule 2.—Use the following Table of Actual Apex Distances as directed.

Remark.—For the following idea, and table, p. 38, we are indebted to Mr. N. F. Jones, Civ. Eng., whose experience in locating gives great weight to his suggestions; some of which have been incorporated in our rules for curves, without special acknowledgment.

$$\begin{array}{c}\text{Apex distance}\\ \text{required,}\end{array} = \begin{array}{c}\text{Apex dist. in table p. 38}\\ \text{for given total angle}\end{array} \times \frac{\text{given radius}}{5730;}$$

$$= \begin{array}{c}\text{(approxi-}\\ \text{mately)}\end{array} = \frac{\text{Apex dist. in table, p. 38 for given total ang.}}{\text{chord deflection angle of given curve, in degrees and decimals.}}$$

(See following "Remark.")

If the central angle exceeds 180°, take it from 360°; call the remainder the central (or substitute) angle, and proceed as above.

Example.—What is the apex or tangent distance $t\,a$ or $t\,z$, Fig. 7, p. 28, for a curve $a\,i\,z$ of 3° 17′ chord-deflection angle; and with a central angle $a\,c\,z$ of 88° 10′?

By the first formula. In table p. 19 we find the radius corresponding to a chord-deflection angle of 3° 17′ is 1745.35 feet. In table p. 40, opposite to total central angle 88° 10′, we find 5550. Hence

$$\begin{array}{c}\text{Apex distance}\\ \text{required}\end{array} = 5550 \times \frac{1745.35}{5730} = 1690.52 \text{ feet.}$$

By the second (approximate) formula. From table below we find that 3° 17′ = 3.°2833. Hence

$$\begin{array}{c}\text{Apex distance}\\ \text{required}\end{array} = \frac{5550}{3.2833} = 1690.37 \text{ feet.}$$

The apex distance given by the table for any total central angle is the actual tangent of *half* that angle for a curve of radius 5730 feet.

Apex distances for total **angles intermediate between those given in the table** may be obtained by simple proportion; thus:

To obtain the actual apex distance for a 4° curve of 55° 32′ deflection. Here, in the table opposite 55° 30′ find 3015, and for 55° 40′ find 3025, the difference being 10; $\frac{2}{10}$ths of which added to 3015 = 3017, the actual apex dist. of a 1° curve of 55° 32′ deflection, which, divided by 4 = 754, the apex distance required.

If either the central or its substitute angle is between 170° and 180°, use Rule 1, p. 36.

Remark.—If the given chord-deflection angle contains such a number of minutes as cannot be *mentally* reduced to decimals of a degree for dividing by it, use the short table below for making the reduction.

Table of Minutes converted into Decimals of a Degree.

Min.	Deg.	Min.	Deg.	Min.	Deg.	Min.	Deg.
1	·016666	16	·266666	31	·516666	46	·766666
2	·033333	17	·283333	32	·533333	47	·783333
3	·050000	18	·300000	33	·550000	48	·800000
4	·066666	19	·316666	34	·566666	49	·816666
5	·083333	20	·333333	35	·583333	50	·833333
6	·100000	21	·350000	36	·600000	51	·850000
7	·116666	22	·366666	37	·616666	52	·866666
8	·133333	23	·383333	38	·633333	53	·883333
9	·150000	24	·400000	39	·650000	54	·900000
10	·166666	25	·416666	40	·666666	55	·916666
11	·183333	26	·433333	41	·683333	56	·933333
12	·200000	27	·450000	42	·700000	57	·950000
13	·216666	28	·466666	43	·716666	58	·966666
14	·233333	29	·483333	44	·733333	59	·983333
15	·250000	30	·500000	45	·750000	60	1·000000

To reduce minutes to decimals of a degree divide them by 60.

TABLE OF ACTUAL APEX DISTANCES—Continued.

For a Curve of 5730 ft. Radius.

Cen. Ang.		Apex D.	Cen. Ang.		Apex D.	Cen. Ang.		Apex D.	Cen. Ang.		Apex D.
°	′		°	′		°	′		°	′	
65		3651	73		4240	81		4894	89		5631
	10	3662		10	4253		10	4908		10	5647
	20	3674		20	4266		20	4923		20	5664
	30	3686		30	4279		30	4938		30	5680
	40	3698		40	4292		40	4952		40	5697
	50	3709		50	4305		50	4966		50	5713
66		3721	74		4318	82		4981	90		5730
	10	3733		10	4331		10	4995		10	5747
	20	3745		20	4344		20	5010		20	5763
	30	3757		30	4357		30	5025		30	5780
	40	3769		40	4370		40	5040		40	5797
	50	3781		50	4383		50	5054		50	5814
67		3793	75		4397	83		5069	91		5831
	10	3805		10	4410		10	5084		10	5848
	20	3817		20	4424		20	5099		20	5865
	30	3829		30	4437		30	5114		30	5882
	40	3841		40	4450		40	5129		40	5899
	50	3853		50	4463		50	5144		50	5916
68		3865	76		4477	84		5159	92		5933
	10	3877		10	4490		10	5174		10	5951
	20	3889		20	4504		20	5190		20	5968
	30	3902		30	4517		30	5205		30	5985
	40	3914		40	4531		40	5220		40	6003
	50	3926		50	4544		50	5235		50	6021
69		3938	77		4558	85		5250	93		6038
	10	3950		10	4571		10	5266		10	6056
	20	3963		20	4585		20	5281		20	6073
	30	3975		30	4599		30	5297		30	6091
	40	3988		40	4613		40	5312		40	6109
	50	4000		50	4626		50	5328		50	6127
70		4012	78		4640	86		5343	94		6145
	10	4025		10	4654		10	5359		10	6163
	20	4037		20	4668		20	5375		20	6181
	30	4049		30	4681		30	5390		30	6199
	40	4062		40	4695		40	5406		40	6217
	50	4075		50	4709		50	5422		50	6235
71		4087	79		4723	87		5438	95		6253
	10	4100		10	4738		10	5453		10	6271
	20	4112		20	4752		20	5469		20	6290
	30	4125		30	4766		30	5485		30	6308
	40	4138		40	4780		40	5501		40	6326
	50	4150		50	4794		50	5517		50	6345
72		4163	80		4808	88		5533	96		6364
	10	4176		10	4822		10	5550		10	6383
	20	4189		20	4837		20	5566		20	6401
	30	4201		30	4851		30	5582		30	6420
	40	4214		40	4865		40	5598		40	6439
	50	4227		50	4880		50	5614		50	6458

TABLE OF ACTUAL APEX DISTANCES—Continued.

For a Curve of 5730 ft. Radius.

Cen. Ang.	Apex D.	Cen. Ang.	Apex D.	Cen. Ang.	Apex D.	Cen. Ang.	Apex D.
° ′		° ′		° ′		° ′	
97	6477	105	7467	113	8657	121	10128
10	6496	10	7490	10	8684	10	10162
20	6515	20	7513	20	8712	20	10197
30	6534	30	7536	30	8740	30	10232
40	6553	40	7558	40	8768	40	10267
50	6572	50	7581	50	8796	50	10302
98	6592	106	7604	114	8824	122	10337
10	6611	10	7627	10	8852	10	10373
20	6630	20	7650	20	8880	20	10408
30	6650	30	7674	30	8908	30	10444
40	6670	40	7697	40	8937	40	10481
50	6689	50	7720	50	8966	50	10517
99	6709	107	7744	115	8994	123	10553
10	6729	10	7767	10	9023	10	10590
20	6749	20	7791	20	9052	20	10627
30	6768	30	7815	30	9081	30	10664
40	6788	40	7839	40	9111	40	10701
50	6808	50	7863	50	9140	50	10739
100	6829	108	7887	116	9170	124	10777
10	6849	10	7911	10	9200	10	10814
20	6869	20	7935	20	9230	20	10853
30	6890	30	7960	30	9260	30	10891
40	6910	40	7984	40	9289	40	10929
50	6930	50	8008	50	9320	50	10968
101	6951	109	8033	117	9351	125	11007
10	6972	10	8058	10	9381	10	11046
20	6992	20	8083	20	9412	20	11086
30	7013	30	8108	30	9442	30	11125
40	7034	40	8133	40	9474	40	11165
50	7055	50	8158	50	9505	50	11205
102	7076	110	8183	118	9536	126	11246
10	7097	10	8209	10	9568	10	11286
20	7118	20	8234	20	9599	20	11327
30	7140	30	8260	30	9631	30	11368
40	7161	40	8286	40	9663	40	11409
50	7182	50	8311	50	9695	50	11451
103	7204	111	8337	119	9728	127	11493
10	7225	10	8364	10	9760	10	11535
20	7247	20	8390	20	9793	20	11577
30	7269	30	8416	30	9825	30	11619
40	7290	40	8442	40	9858	40	11662
50	7312	50	8468	50	9891	50	11705
104	7334	112	8495	120	9925	128	11748
10	7356	10	8522	10	9958	10	11792
20	7378	20	8549	20	9992	20	11835
30	7400	30	8576	30	10025	30	11880
40	7423	40	8603	40	10059	40	11924
50	7445	50	8630	50	10093	50	11968

Example.—Let the radius be 2865 feet, and the chord 100 feet; what will be the tangential and deflection angles?

Here, half the chord (50 feet) divided by the radius (2865 feet), gives ·01745; and the tangential angle in the table corresponding to the natural sine ·01745 is 1°, twice which is 2°, the deflection angle required.

Rule 2.—The deflection angle for 100 feet chords may be found **approximately** (see first foot-note, p. 35) by dividing 5730 by the radius. This is very close for curves of over 500 feet radius. For 500 feet it gives about one minute too little. Or, 343800 divided by the radius will give the deflection angle in minutes; and these divided by 60 will give the angle in degrees and minutes.

Proof.—5730 feet is the radius of a one degree curve; and 343800 feet is the radius of a one minute curve; and the deflection angles of curves are inversely as their radii, approximately.

Example 1.—What is the deflection angle for a radius of 2865 feet, the chords being 100 feet each?

Here, 5730 divided by the radius 2865, gives 2°, the deflection angle required.

Example 2.—What is the deflection angle for radius 2022 feet; chords 100 feet?

Here, 5730 divided by 2022 gives the deflection angle 2°·833, or partly in *decimals* of a degree. Now, in the table on page 37, we see that the decimal ·833 of a degree is 50 minutes. Therefore, the required angle is 2° 50′, as in our Table of Radii, etc., p. 18.

Example 3.—What is the deflection angle for a radius of 969 feet?

Here, 343800 divided by 969 gives 355 minutes; and $\frac{355}{60} = 5° 55'$, the required angle.

Rule 3.—Having only the apex distance, and the total central angle. First find the radius thus: Divide the apex distance by the tangent of *half* the central angle. Then use Rule 1 of this Article, or Table of Radii, p. 18.

Rule 4.—For a track already laid, measure at the rails a chord of 100 feet, and its middle ordinate in feet. Refer to our Table of Ordinates, p. 50, for the deflection angle.

For sub-tangential and sub-deflection angles, see Arts. IV. and V.

ARTICLE XIII.

Deflection Distances.

To find the Deflection Distance (exactly) for any given radius, and for equal chords of any given length.

Rule I.—As the radius : the chord : : the chord : the deflection distance. *Or, in other words,* Divide the square of the chord in feet by the radius in feet.

Remark.—For chords of 100 feet, Rule 1 becomes: Divide 10000 by the radius in feet.

(The deflection distance to a radius of 10000 feet, with chords of 100 feet, is 1 foot; and the deflection distances for other radii increase *inversely* as the radii.)

Example.—What is the deflection distance for a radius of 5730 feet, the chords being 100 feet long?

Here, 10000 divided by 5730 radius, gives 1·745 feet, the deflection distance required.

Rule 2.—Divide half the given chord by radius; the quotient will be the natural sine of one-*half* the deflection angle. Multiply *double* this natural sine by the chord. The product will be the deflection distance required. By this rule our table was prepared.

Example.—As before, what is the deflection distance to a radius of 5730 feet, the chords being 100 feet long?

Here, half the chord (50 feet) divided by radius (5730 feet) gives ·008726, which is the natural sine of half the deflection angle. Now ·008726 multiplied by 2, gives ·017452, which, multiplied by the chord (100 feet), gives 1·745 feet, the required deflection distance, the same as in the preceding example.

ARTICLE XIV.

Tangential Distances.

To find (exactly) the Tangential Distance corresponding to any given radius, and to equal chords of any given length.

Rule.—First find the tangential angle by Art. XII., and take from the table of natural sines, that corresponding to one-*half* of the *tangential* angle. Then multiply *double* this sine by the given chord, for the tangential distance. By this rule our table was prepared.

Example.—Let the radius be 2865 feet, and the chords 100 feet each; what will be the tangential distance?

Here, we find, by Art. XII., the tangential angle 1° for a radius of 2865 feet.

The natural sine corresponding to 30 minutes, or one-half of this tangential angle, is, by the table of sines, ·008727; the double of which is ·017454, which, multiplied by the chord, or 100 feet, gives 1·745 feet for the tangential distance required.

ARTICLE XV.

Ordinates.

To find the Middle Ordinate to any given radius, and to any given chord.

Rule I.—From the square of the radius subtract the square of *half* the chord. Find the square root of the remainder. Take this square root from the radius.

Example.—What is the length of the middle ordinate *d e*, Fig. 13, the radius *c a* being 819 feet, and the chord *a b* 100 feet?

Here, the square of *c a* (819) is 670761, and the square of *a e* (50) is 2500; which, being subtracted from the former, leaves 668261; the square root of which is *e c*, 817·472; which, taken from the radius 819, leaves 1·528 feet, the required middle ordinate *d e*.

Rule 2.—With any chord the middle ordinate is equal to the radius multiplied by nat versed sine of *half* the angle subtended by the chord.

With chords of 100 feet this becomes: Middle ordinate is equal to radius multiplied by nat versed sine of tangential angle.

Rule 3.—With any chord the middle ordinate is equal to *half* the chord multiplied by nat tangent of *quarter* the angle subtended by the chord.

With chords of 100 feet this becomes: Middle ordinate is equal to *half* the chord multiplied by nat tangent of quarter the deflection angle.

Rule 4.—Approximate. With radius 500 feet or more, divide the square of half the chord by the *diameter* of the curve. When the chord is 100 feet this becomes: $\frac{2500}{\text{Twice radius}}$. With 300 feet radius this rule gives ·030 too short in 4·197 feet; and with 100 feet radius ·897 short in 13·397. See also Rule 2, next Article.

Rule 5.—For 100 ft. chords. Approximate. With radius 500 feet or more, divide the tangential distance by 4.

With 300 ft. radius this rule gives but ·013 of a foot too short in 4·197 feet; and with 100 radius ·456 too short in 13·398.

ARTICLE XVI.

Having given the Middle Ordinate d e, *Fig.* 13, *it is required to find any other one, as* i n.

Rule 1.—Subtract the middle ordinate *d e* from the radius *d c;* the remainder will be *e c:* then from the square of the radius *c i* subtract the square of the distance *o i,* which the required ordinate *i n* is from the middle ordinate *d e,* and extract the square root of the remainder. This square root will be *o c.* From this square root *o c* subtract *e c;* the remainder will be *o e,* which is equal to *i n* the required ordinate.

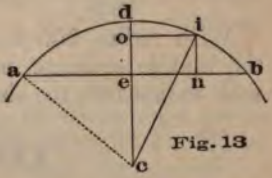


Fig. 13

Example.—The middle ordinate *d e,* of a 100 ft. chord *b a,* to a radius of 819, being 1·528 feet, it is required to find the length of the ordinate *i n,* 20 feet from the middle one.

Here, the middle ordinate *d e,* 1·528, subtracted from the radius 819, leaves *e c,* 817·472. The square of the radius is 670761; and the square of 20 (the distance of the required ordinate from the middle one) is 400; which, taken from 670761, leaves 670361; the square root of which is 818·756, or *o c;* from which take *e c,* or 817·472, and the remainder, 1·284, will be *o e,* which is equal to *i n,* the required ordinate.

Rule 2.—Approximate. With radius not less than 500 feet, chords 100 feet, multiply *any* ordinate of a 1° curve (Table, p. 50) by the chord-deflection angle (in degrees and decimals, table, p. 37) of the new curve. The product will be the corresponding new ordinate.

For 300 ft. radius this rule gives middle ordinate ·013 foot too short in 4·197 feet; and for 100 radius ·318 too short in 13·398 feet.

Rule 3.—Any ordinate for chords not exceeding 100 feet in length, and for radii not less than 500 feet, may be found near enough for practice, thus: Divide the rectangle of the segments of the chord by the *diameter* of the curve.

Example.—What is the ordinate at 15 feet from the end of the 100 feet chord, for a radius of 819 feet?

Here, the ordinate divides the chord into 2 segments, of 15 and 85 feet. The rectangle of these, or 15 × 85, is 1275. The radius being 819, the diameter is 1638; consequently $\frac{1275}{1638} = .778$ feet, the required ordinate.

With radius 300 feet, chord 100 feet, this rule makes the middle ordinate only ·030 too short in 4·197; and with radius 100, too short ·897 in 13·397 feet.

ARTICLE XVII.

To find Ordinates for Sub-Chords.

These must be calculated as they are needed. It would not be possible to give tables for all supposable cases. They may be found approximately enough *for railroad practice*, for curves of over 300 ft. radius, and for chords not exceeding 100 feet, thus:

The sub-chord being supposed to be divided into the same number of equal parts as the whole chord, then in any circle of given radius, not less than about 300 feet, the ordinates of an entire 100 ft. chord may be assumed to be to those of a sub-chord, as the square of the chord is to the square of the sub-chord.

In all our tables the chord is supposed to be 100 feet, the square of which is 10000; the rule therefore becomes, as 10000 feet : square of sub-chord in feet :: Ord. of Chord : Ord. of Sub-chord *approximately*.

Example.—In a curve of 5730 ft. radius, the middle ordinate of a 100 ft. chord is ·218 of a foot; what will be the length of the middle ordinate of a sub-chord of 50 feet? Here,

Sq. of 100 ft.	:	Sq. of 50 ft.	::	Mid. Ord. of Chord.	:	Mid. Ord. Sub-Chord *approximately.*
10000	:	2500	::	·218 ft.	:	·0545 ft.

Or, they may be found for any radius, thus:

Suppose we need the ordinates for a sub-chord 46 feet long; radius 1810 feet; or chord-deflection angle 3° 10′.

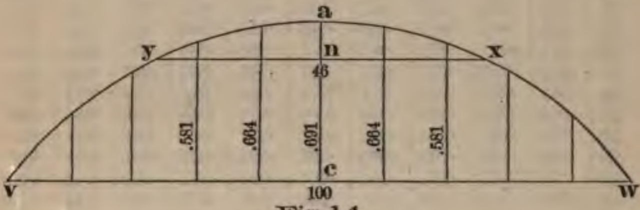


Fig. 14

Make a rough pencil sketch, Fig. 14 (which need not be to a scale), of a 100 ft. chord *v w*, with either all its ordinates for 1810 ft. rad., 5 feet apart (taken from Table, p. 50), or with only alternate ones 10 feet apart, as in the Fig., and set down their lengths. Also draw the given sub-chord *y x* across as many of the ordinates as its length requires. In this instance of course across 5 of them.

By one of the rules in Art. XV. find the middle ordinate *a n* (·146) of the sub-chord. Subtract it from the middle ordinate *a c*, or ·691. The remainder (·545) will be *c n*. Now it is self-evident that this ·545 taken from ·664 and ·581 gives the other four sub-ordinates ·119 and ·036; two of each.

Consequently we have found 5 sub-ordinates for the sub-chord *y x*. The distances between them must plainly be laid off each way *from the middle* of the sub-chord, instead of from the ends as usual.

TABLE OF ORDINATES (*in Feet*).

Ordinates five feet apart.—Chord one hundred feet.

Distances of the Ordinates from the end of the 100 feet Chord.										
Angle of Defl'n.	Middle, 50 feet.	45 feet.	40 feet.	35 feet.	30 feet.	25 feet.	20 feet.	15 feet.	10 feet.	5 feet.
° ′										
2	·007	·007	·007	·006	·006	·005	·003	·003	·002	·001
4	·014	·014	·014	·013	·012	·010	·008	·008	·005	·003
6	·022	·021	·021	·020	·019	·016	·013	·011	·008	·004
8	·029	·029	·028	·026	·024	·022	·018	·015	·010	·005
10	·036	·036	·035	·033	·031	·027	·023	·019	·013	·007
12	·043	·043	·041	·038	·037	·033	·028	·022	·015	·008
14	·051	·050	·048	·044	·043	·038	·032	·026	·017	·010
16	·058	·058	·056	·052	·049	·044	·037	·030	·020	·011
18	·065	·065	·063	·059	·055	·050	·042	·033	·023	·013
20	·073	·072	·070	·066	·061	·055	·047	·037	·026	·014
22	·080	·079	·076	·071	·067	·060	·051	·041	·029	·015
24	·087	·086	·083	·077	·074	·066	·056	·045	·031	·017
26	·095	·093	·090	·084	·080	·071	·060	·048	·034	·018
28	·102	·101	·098	·092	·086	·077	·065	·052	·036	·019
30	·109	·108	·105	·099	·092	·082	·070	·055	·039	·020
32	·116	·115	·112	·106	·098	·088	·075	·058	·042	·022
34	·123	·122	·118	·111	·104	·094	·079	·062	·044	·023
36	·131	·130	·126	·119	·110	·099	·084	·066	·047	·024
38	·138	·137	·133	·126	·116	·105	·089	·070	·049	·025
40	·145	·144	·140	·133	·123	·110	·093	·074	·052	·027
42	·153	·150	·146	·138	·128	·115	·098	·077	·055	·028
44	·160	·158	·153	·145	·135	·121	·103	·081	·057	·030
46	·167	·165	·160	·152	·141	·126	·107	·085	·060	·032
48	·174	·172	·167	·158	·147	·132	·112	·088	·062	·033
50	·182	·180	·175	·166	·153	·138	·117	·092	·065	·034
52	·189	·187	·181	·171	·159	·143	·122	·095	·068	·035
54	·197	·194	·188	·178	·165	·148	·126	·099	·070	·036
56	·204	·202	·195	·185	·171	·154	·131	·103	·073	·038
58	·211	·209	·202	·192	·177	·159	·136	·107	·075	·039
1°	·218	·216	·209	·198	·183	·164	·140	·111	·078	·041
2	·225	·223	·215	·204	·189	·169	·145	·114	081	·042
4	·233	·231	·223	·211	·196	·175	·150	·118	083	·043
6	·240	·238	·230	·217	202	180	·155	·121	·086	·045
8	·247	·245	·237	·224	·208	·186	·159	·125	·088	·046
10	·255	·252	·244	·231	·214	·191	·163	·130	·091	·048
12	·262	·260	·252	·237	·220	·196	·168	·133	·094	·049
14	·269	·267	·258	·244	·226	·202	·173	·136	·096	·050
16	·276	·274	·265	·251	·232	·207	·177	·140	·099	·052
18	·284	·282	·273	·257	·238	·213	·182	·144	·101	·053
20	·291	·288	·279	·264	·244	·218	·187	·148	·104	·055

TABLE OF ORDINATES (*in Feet*)—Continued.

Ordinates five feet apart.—Chord one hundred feet.

Distances of the Ordinates from the end of the 100 feet Chord.										
Angle of Defl'n.	Middle, 50 feet.	45 feet.	40 feet.	35 feet.	30 feet.	25 feet.	20 feet.	15 feet.	10 feet.	5 feet.
° ′										
1 22	·298	·295	·285	·270	·250	·224	·192	·151	·107	·056
24	·306	·303	·293	·277	·256	·229	·197	·155	·109	·057
26	·313	·310	·300	·284	·263	·235	·201	·159	·112	·059
28	·320	·317	·307	·291	·269	·240	·206	·163	·114	·060
30	·327	·324	·314	·297	·275	·246	·210	·167	·117	·062
32	·334	·331	·321	·304	·281	·251	·215	·171	·120	·063
34	·341	·338	·328	·310	·287	·257	·219	·174	·122	·065
36	·349	·345	·335	·317	·293	·262	·224	·178	·125	·066
38	·356	·353	·342	·323	·299	·268	·228	·182	·127	·068
40	·364	·360	·349	·330	·305	·273	·233	·185	·130	·069
42	·371	·367	·356	·337	·312	·278	·238	·189	·133	·070
44	·378	·374	·363	·343	·318	·284	·242	·192	·135	·072
46	·385	·382	·370	·350	·324	·289	·247	·196	·138	·073
48	·393	·389	·377	·356	·330	·295	·251	·200	·141	·075
50	·400	·396	·384	·364	·336	·300	·256	·204	·144	·076
52	·407	·403	·391	·370	·342	·305	·261	·208	·147	·077
54	·414	·410	·398	·376	·348	·311	·265	·211	·149	·079
56	·422	·418	·405	·383	·354	·316	·270	·215	·152	·080
58	·429	·425	·412	·389	·360	·322	·275	·219	·154	·082
2°	·436	·432	·419	·397	·366	·327	·280	·222	·157	·083
2	·443	·439	·426	·402	·373	·332	·284	·226	·160	·084
4	·451	·446	·433	·409	·379	·338	·289	·230	·162	·086
6	·458	·454	·440	·416	·385	·343	·293	·234	·165	·087
8	·465	·461	·447	·425	·391	·349	·298	·237	·167	·088
10	·473	·468	·454	·430	·397	·355	·303	·241	·170	·089
12	·480	·475	·461	·437	·403	·360	·308	245	·173	·090
14	·487	·482	·468	·443	·409	·366	·312	·248	·175	·092
16	·495	·490	·475	·450	·415	·371	·317	·252	·178	·093
18	·502	·497	·482	·456	·421	·377	·321	·256	·180	·095
20	·509	·504	·489	·463	·428	·382	·326	·260	·183	·096
22	·516	·511	·496	·470	·434	·387	·330	·264	·186	·097
24	·523	·518	·503	·476	·440	·393	·334	·267	·188	·099
26	·531	·526	·510	·483	·446	·398	·338	·271	·191	·100
28	·538	·533	·517	·489	·452	·404	·346	·275	·194	·102
30	·545	·540	·524	·496	·458	·409	·350	·278	·196	·103
32	·552	·547	·531	·503	·465	·415	·355	·282	·199	·104
34	·560	·554	·538	·509	·471	·420	·359	·285	·201	·106
36	·567	·562	·545	·516	·477	·425	·364	·289	·204	·107
38	·574	·569	·552	·522	·483	·431	·368	·293	·206	·109
40	·582	·576	·559	·529	·489	·436	·373	·297	·209	·110

TABLE OF ORDINATES (*in Feet*)—Continued.

Ordinates five feet apart.—Chord one hundred feet.

	Distances of the Ordinates from the end of the 100 feet Chord.									
Angle of Defl'n.	Middle, 50 feet.	45 feet.	40 feet.	35 feet.	30 feet.	25 feet.	20 feet.	15 feet.	10 feet.	5 feet.
° ′										
2 42	·589	·583	·566	·536	·495	·441	·378	·301	·212	·111
44	·596	·590	·573	·542	·501	·447	·382	·304	·214	·113
46	·603	·598	·580	·549	·507	·452	·387	·308	·217	·114
48	·611	·605	·587	·555	·513	·458	·391	·312	·219	·116
50	·618	·612	·594	·562	·519	·464	·396	·315	·222	·117
52	·625	·619	·601	·569	·526	·469	·401	·319	·225	·118
54	·632	·626	·608	·575	·532	·474	·405	·322	·227	·119
56	·640	·634	·615	·582	·538	·480	·410	·326	·230	·121
58	·647	·641	·622	·588	·544	·485	·414	·330	·232	·123
3°	·654	·648	·629	·595	·550	·491	·419	·334	·235	·124
2	·661	·655	·636	·602	·556	·496	·424	·338	·238	·125
4	·669	·662	·643	·608	·562	·502	·428	·341	·240	·127
6	·676	·670	·650	·615	·568	·507	·433	·345	·243	·128
8	·683	·677	·657	·621	·574	·512	·438	·349	·246	·130
10	·691	·684	·664	·629	·581	·518	·443	·353	·249	·131
12	·698	·691	·671	·635	·587	·523	·448	·357	·251	·132
14	·706	·698	·678	·642	·593	·529	·452	·360	·254	·134
16	·713	·705	·685	·649	·599	·534	·457	·364	·257	·135
18	·720	·713	·692	·655	·605	·540	·462	·368	·259	·137
20	·727	·720	·699	·662	·611	·545	·466	·371	·262	·138
22	·734	·727	·706	·668	·617	·550	·471	·375	·264	·139
24	·742	·734	·713	·675	·623	·556	·475	·378	·267	·141
26	·749	·742	·720	·682	·629	·561	·480	·382	·270	·142
28	·756	·749	·727	·688	·635	·567	·485	·386	·272	·144
30	·764	·756	·734	·695	·642	·573	·489	·390	·275	·145
32	·771	·763	·741	·702	·648	·578	·494	·394	·278	·146
34	·779	·770	·748	·708	·654	·584	·498	·397	·280	·148
36	·786	·777	·755	·715	·660	·589	·503	·401	·283	·149
38	·793	·785	·762	·721	·666	·594	·508	·405	·285	·151
40	·800	·792	·769	·728	·673	·600	·512	·408	·288	·152
42	·807	·799	·776	·734	·679	·605	·517	·412	·291	·153
44	·814	·806	·783	·741	·685	·611	·521	·415	·293	·155
46	·822	·814	·790	·748	·691	·616	·526	·419	·296	·156
48	·829	·821	·797	·754	·697	·621	·531	·423	·298	·158
50	·836	·828	·804	·761	·703	·627	·536	·427	·301	·159
52	·843	·835	·811	·768	·709	·632	·541	·431	·304	·160
54	·850	·842	·818	·774	·715	·638	·545	·434	·306	·162
56	·858	·850	·825	·781	·721	·643	·550	·438	·309	·163
58	·865	·857	·832	·787	·728	·648	·555	·442	·311	·165
4°	·873	·864	·839	·794	·734	·655	·559	·445	·314	·166

TABLE OF ORDINATES (*in Feet*)—Continued.

Ordinates five feet apart.—Chord one hundred feet.

	Distances of the Ordinates from the end of the 100 feet Chord.									
Angle of Defl'n.	Middle, 50 feet.	45 feet.	40 feet.	35 feet.	30 feet.	25 feet.	20 feet.	15 feet.	10 feet.	5 feet.
° '										
4 5	·891	·882	·856	·810	·749	·668	·571	·454	320	·169
10	·909	·900	·874	·827	·764	·682	·582	·464	·327	·173
15	·927	·918	·891	·844	·780	·695	·594	·473	·334	·176
20	·945	·936	·909	·860	·795	·709	·606	·482	·340	·179
25	·963	·954	·926	·877	·810	·723	·617	·491	347	·183
30	·981	·972	·944	·893	·825	·736	·629	·501	354	·186
35	·999	·990	·961	·909	·840	·750	·640	·510	360	·189
40	1·017	1·008	·979	·926	·855	·764	·652	·519	·367	·193
45	1·036	1·026	·996	·943	·871	·777	·664	·529	·373	·196
50	1·054	1·044	1·014	·959	·886	·791	·676	·538	·380	·199
55	1·072	1·062	1·031	·976	·901	·804	·687	·547	·386	·203
5°	1·091	1·080	1·048	·993	·917	·818	·699	·557	·393	·207
5	1·109	1·098	1·065	1·009	·932	·831	·711	·566	·400	·210
10	1·127	1·116	1·083	1·026	·947	·845	·722	·576	·406	·214
15	1·146	1·134	1·100	1·042	·963	·859	·734	·585	·413	·217
20	1·164	1·152	1·118	1·058	·978	·872	·746	·594	·419	·220
25	1·182	1·170	1·135	1·075	·993	·886	·757	·603	·426	·224
30	1·200	1·188	1·153	1·092	1·009	·900	·769	·613	·432	·228
35	1·218	1·206	1·170	1·108	1·024	·913	·781	·622	·438	·231
40	1·236	1·224	1·188	1·124	1·039	·927	·792	·631	·445	·235
45	1·255	1·242	1·205	1·141	1·055	·941	·804	·640	·452	·238
50	1·273	1·260	1·223	1·157	1·070	·954	·816	·649	·458	·241
55	1·291	1·278	1·240	1·174	1·085	·967	·827	·658	·465	·245
6°	1·309	1·296	1·258	1·191	1·100	·982	·839	·668	·472	·248
5	1·327	1·314	1·275	1·207	1·115	·995	·851	·677	·478	·251
10	1·345	1·332	1·293	1·224	1·130	1·009	·862	·686	·485	·255
15	1·364	1·350	1·310	1·240	1·146	1·023	·874	·696	·492	·259
20	1·382	1·368	1·328	1·256	1·161	1·036	·886	·705	·498	·262
25	1·400	1·386	1·345	1·273	1·176	1·050	·897	·714	·505	·266
30	1·419	1·404	1·362	1·290	1·192	1·064	·909	·724	·511	·269
35	1·437	1·422	1·379	1·306	1·207	1·077	·921	·733	·517	·272
40	1·455	1·440	1·397	1·323	1·222	1·091	·932	·742	·524	·276
45	1·473	1·458	1·415	1·339	1·238	1·105	·944	·752	·531	·280
50	1·491	1·476	1·432	1·355	1·253	1·118	·956	·761	·537	·283
55	1·509	1·494	1·450	1·372	1·268	1·132	·967	·770	·544	·287
7°	1·528	1·512	1·467	1·389	1·284	1·146	979	·779	·551	290
5	1·546	1·530	1·484	1·405	1·299	1·159	·991	·788	·557	·293
10	1·564	1·548	1·502	1·422	1·314	1·173	1·002	·798	·564	297
15	1·582	1·566	1·520	1·438	1·330	1 187	1·014	·807	·570	301
20	1·600	1·584	1·537	1·454	1·345	1·206	1·026	·816	·576	·304

TABLE OF ORDINATES (*in Feet*)—Continued.

Ordinates five feet apart.—Chord one hundred feet.

Distances of the Ordinates from the end of the 100 feet Chord.										
Angle of Defl'n.	Middle 50 feet.	45 feet.	40 feet.	35 feet.	30 feet.	25 feet.	20 feet.	15 feet.	10 feet.	5 feet.
7° 25′	1·618	1·602	1·555	1·471	1·360	1·214	1·037	·825	·583	·308
30	1·637	1·620	1·572	1·488	1·375	1·228	1·048	·835	·590	·311
35	1·655	1·638	1·589	1·504	1·390	1·241	1.060	·844	·596	·314
40	1·673	1·656	1·607	1·521	1·405	1·255	1·071	·854	·603	·318
45	1·692	1·674	1·624	1·537	1·421	1·269	1·083	·863	·610	·321
50	1·710	1·692	1·641	1·553	1·436	1·282	1·095	·872	·616	·324
55	1·728	1·710	1·659	1·570	1·451	1·296	1·106	·881	·623	·328
8°	1·746	1·728	1·677	1·587	1·467	1·310	1·118	·891	·629	·332
15	1·801	1·782	1·729	1·637	1·513	1·351	1·153	·918	·649	·342
30	1·855	1·836	1·782	1·687	1·559	1·392	1·188	·946	·669	·353
45	1·910	1·890	1·834	1·737	1·605	1·433	1·223	·974	·689	·363
9°	1·965	1·944	1·886	1·787	1·651	1·474	1·258	1·002	·708	·373
15	2·019	1·998	1·939	1·837	1·696	1·515	1·293	1·030	·728	·384
30	2·074	2·052	1·991	1·887	1·742	1·556	1·328	1·057	·748	·394
45	2·128	2·106	2·044	1·937	1·788	1·597	1·363	1·085	·767	·405
10°	2·183	2·161	2·096	1·987	1·834	1·637	1·398	1·114	·787	·415
15	2·238	2·215	2·148	2·037	1·880	1·678	1·433	1·142	·807	·425
30	2·292	2·269	2·201	2·087	1·926	1·719	1·468	1·170	·827	·436
45	2·347	2·323	2·254	2·136	1·972	1·761	1·503	1·198	·846	·446
11°	2·401	2·377	2·306	2·186	2·018	1·802	1·538	1·226	·866	·457
15	2·456	2·432	2·359	2·236	2·064	1·843	1·574	1·254	·886	·467
30	2·511	2·486	2·411	2·286	2·110	1·884	1·609	1·282	·906	·478
45	2·566	2·540	2·464	2·336	2·156	1·926	1·644	1·310	·926	·488
12°	2·620	2·594	2·516	2·386	2·203	1·967	1·680	1·339	·946	·499
15	2·675	2·649	2·569	2·436	2·249	2·008	1·715	1·367	·966	·509
30	2·730	2·703	2·621	2·485	2·295	2·049	1·750	1·395	·985	·520
45	2·785	2·757	2·674	2·535	2·341	2·091	1·785	1·423	1·005	·530
13°	2·839	2·811	2·726	2·585	2·387	2·132	1·820	1·451	1·025	·541
15	2·894	2·865	2·779	2·635	2·433	2·173	1·855	1·479	1·045	·551
30	2·949	2·920	2·832	2·685	2·479	2·214	1·891	1·507	1·065	·562
45	3·003	2·974	2·884	2·735	2·525	2·256	1·926	1·535	1·085	·572
14°	3·058	3·028	2·937	2·785	2·571	2·297	1·961	1·564	1·105	·583
15	3·113	3·082	2·989	2·834	2·618	2·338	1·996	1·592	1·124	·593
30	3·168	3·136	3·042	2·884	2·664	2·379	2·031	1·620	1·144	·604
45	3·222	3·191	3·094	2·934	2·710	2·421	2·067	1·648	1·164	·614
15°	3·277	3·245	3·147	2·984	2·756	2·462	2·102	1·676	1·184	·625
15	3·332	3·299	3·200	3·034	2·802	2·503	2·137	1·704	1·204	·635
30	3·387	3·354	3·252	3·084	2·848	2·544	2·172	1·732	1·224	·646
45	3·442	3·408	3·305	3·134	2·895	2·586	2·208	1·760	1·244	·656
16°	3·496	3·462	3·358	3·184	2·941	2·627	2·243	1·789	1·264	·667

TABLE OF ORDINATES (*in Feet*)—Continued.

Ordinates five feet apart.—Chord one hundred feet.

Distances of the Ordinates from the end of the 100 feet Chord.										
Angle of Defl'n.	Middle, 50 feet.	45 feet	40 feet.	35 feet.	30 feet.	25 feet.	20 feet.	15 feet.	10 feet.	5 feet.
° ′										
16 30	3·606	3·571	3·463	3·284	3·033	2·710	2·314	1·845	1·304	·688
17	3·716	3·680	3·569	3·384	3·125	2·792	2·384	1·902	1·344	·709
30	3·826	3·788	3·674	3·484	3·218	2·875	2·455	1·958	1·384	·730
18	3·935	3·897	3·779	3·584	3·310	2·958	2·525	2·014	1·424	·751
30	4·045	4·006	3·885	3·684	3·403	3·040	2·596	2·071	1·464	·772
19	4·155	4·115	3·990	3·784	3·495	3·123	2·666	2·127	1·504	·793
30	4·265	4·223	4·096	3·884	3·588	3·205	2·737	2·184	1·544	·814
20	4·375	4·332	4·201	3·984	3·680	3·288	2·808	2·240	1·583	·836
21	4·595	4·549	4·412	4·184	3·864	3·454	2·950	2·353	1·663	·879
22	4·815	4·768	4·624	4·386	4·050	3·620	3·093	2·467	1·744	·922
23	5·035	4·986	4·836	4·587	4·237	3·786	3·236	2·581	1·824	·965
24	5·255	5·204	5·048	4·789	4·423	3·952	3·379	2·695	1·905	1·008
25	5·476	5·422	5·260	4·989	4·609	4·119	3·522	2·809	1·986	1·051
26	5·697	5·642	5·473	5·192	4·798	4·286	3·665	2·924	2·068	1·094
27	5·918	5·860	5·685	5·393	4·984	4·454	3·808	3·039	2·150	1·137
28	6·139	6·079	5·898	5·595	5·171	4·622	3·952	3·154	2·232	1·181
29	6·361	6·298	6·110	5·796	5·357	4·790	4·095	3·269	2·314	1·224
30	6·582	6·517	6·323	5·999	5·544	4·958	4·239	3·385	2·396	1·268
31	6·804	6·737	6·537	6·202	5·733	5·127	4·384	3·502	2·481	1·312
32	7·027	6·957	6·751	6·406	5·922	5·297	4·530	3·619	2·565	1·356
33	7·252	7·178	6·965	6·609	6·111	5·467	4·676	3·737	2·649	1·401
34	7·472	7·398	7·179	6·813	6·300	5·637	4·822	3·854	2·733	1·445
35	7·694	7·619	7·393	7·017	6·489	5·807	4·968	3·972	2·817	1·490
36	7·918	7·841	7·609	7·222	6·679	5·978	5·115	4·090	2·901	1 535
37	8·143	8·063	7·825	7·427	6·870	6·149	5·262	4·209	2·985	1·581
38	8·367	8·286	8·041	7·633	7·060	6·320	5·410	4·327	3·069	1·626
39	8·592	8·508	8·257	7·838	7·251	6·491	5·557	4·446	3·153	1·672
40	8·816	8·731	8·474	8·044	7·442	6·663	5·705	4·565	3·238	1.718

ARTICLE XVIII.

On Long Chords.

It is sometimes convenient, in preliminary locations, to lay off curves by chords longer than 100 feet. For instance, in Fig. 15, instead of running from *a* by chords *a b*, *b c*, *c d*, etc., of but 100 feet, points *d*, *f*, *g*, etc., may be obtained with less trouble by using three times the tangential or deflection angles of the table (as the case may be), and employing chords *a d*, *d f*, *f g*, etc., *nearly* three times as long as the chords *a b*, *b c*, etc.; or if *a d*, *d f*, *f g* be either 2 or 4 stations apart, then 2 or 4 times the tangential and deflection angles would be used; and chords nearly 2 or 4 times 100 feet in length.

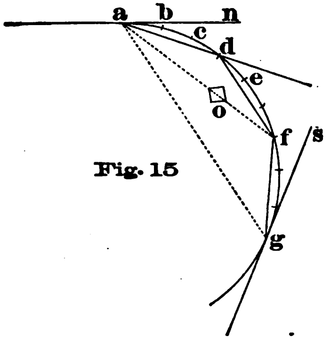


Fig. 15

The next table contains the correct length of chord required to subtend from 1 to 6 stations.

To find the length of a Long Chord to subtend any given number of 100 *ft. chords.*

Rule.—Multiply the *tangential* angle for 100 ft. chords by the given number of 100 ft. chords. The product will be *half* the central angle subtended by the long chord. Find the nat sine of this angle. Multiply this sine by the radius of the curve. The product will be *half* the long chord. Multiply it by 2.

Or, as a formula,

Long Chord=*sine of* (*tangl. angle* × *No. of* 100 *ft. chords*) × *radius* × 2.

To find the Middle Ordinate of any Long Chord, if such should be desired.

Rule.—Multiply the *tangential* angle for 100 ft. chords by the number of 100 ft. chords subtended by the long chord. Find the nat. versed sine of the product. Multiply this versed sine by the radius of the curve.

Remark 1.—*Long chords may at times be useful for passing an obstacle.*

Thus, suppose we are running the curve *a g*, Fig. 15, by tangential angles and chords *a d*, *d f*, *f g*, of 100 feet; the transit being at *a*. An obstacle *o* prevents taking a sight on *f*; but the curve may still be continued from *a* by using a long chord *a g*, subtending the three 100 ft. chords. The angle *n a g* or *s g a* will here equal the sum of the three tangential angles *n a d*, *d a f*, *f a g*.

It is plain that from *a* we may, if necessary, continue the curve beyond *g* by tangential angles and 100 ft. chords as before meeting with the obstacle.

This Method, and those in Arts. I. and XXXVII., will cover nearly all cases in practice for the passing of obstacles.

Remark 2.—With the method of curving by long chords *alone*, the instrument should be moved to each successive point after it is determined, in order to fix the next one, instead of attempting to obtain more than one point from one position of the instrument; because when the chords are longer than one chain they cannot be measured in the right direction by eye, but must be guided by the instrument.

Remark 3.—It must be borne in mind that, in any given curve, only the tangential and deflection *angles* increase in the same proportion as the number of 100 feet stations subtended by the long chord. Therefore, *these* long chords cannot be used for laying out curves *by eye*, as their tangential and deflection *distances* are not here given.

When it is required to use long chords for turning a curve by Art. 3, they should be composed of a number of *whole chains*, being made say, 200, 300, or 400, etc., feet in length, because the *deflection* distances of curves of given radius are *exactly*, and the *tangential* distances *approximately* as the squares of the number of chains in the length of the long chord. For instance, to lay off a 5° curve by chords of 200, 300, or 400 feet in length, the tangential and deflection distances of the table must be multiplied by 4, 9, or 16, as the case may be. In this case the tangential and deflection *angles* are unknown.

TABLE OF LONG CHORDS.

Radius in feet.	Angle of Deflection.	Length of Chord in feet required to subtend					
		1 Station.	2 Stations.	3 Stations.	4 Stations.	5 Stations.	6 Stations.
5729·6	1°	100	200·0	300·0	399·9	499·8	599·7
4583·8	¼	100	200·0	300·0	399·9	499·8	599·6
3819·8	½	100	200·0	299·9	399·8	499·7	599·4
3274·2	¾	100	200·0	299·9	399·8	499·5	599·2
2864·9	2°	100	200·0	299·9	399·7	499·4	598·9
2546·6	¼	100	200·0	299·9	399·6	499·2	598·6
2292·0	½	100	200·0	299·8	399·5	499·0	598·3
2083·7	¾	100	200·0	299·8	399·4	498·8	598·0
1910·1	3°	100	199·9	299·7	399·3	498·6	597·6
1763·2	¼	100	199·9	299·7	399·2	498·4	597·2
1637·3	½	100	199·9	299·6	399·1	498·1	596·7
1528·2	¾	100	199·9	299·6	399·0	497·9	596·2
1432·7	4°	100	199·9	299·5	398·8	497·6	595·7
1348·5	¼	100	199·9	299·5	398·6	497·3	595·2
1273·6	½	100	199·8	299·4	398·5	496·9	594·6
1206·6	¾	100	199·8	299·3	398·3	496·6	594·0
1146·3	5°	100	199·8	299·2	398·1	496·2	593·4
1091·7	¼	100	199·8	299·1	397·8	495·8	592·7
1042·1	½	100	199·8	299·0	397·7	495·4	592·0
996·9	¾	100	199·7	298·9	397·5	495·0	591·2
955·4	6°	100	199·7	298·9	397·3	494·5	590·4
917·2	¼	100	199·7	298·8	397·0	494·1	589·6
881·9	½	100	199·7	298·7	396·8	493·6	588·8
849·3	¾	100	199·6	298·6	396·5	493·1	587·9
819·0	7°	100	199·6	298·5	396·3	492·6	587·0
790·8	¼	100	199·6	298·4	396·0	492·0	586·1
764·5	½	100	199·6	298·3	395·7	491·5	585·1
739·9	¾	100	199·6	298·1	395·4	490·9	584·1
716·8	8°	100	199·5	298·0	395·1	490·3	583·1
695·1	¼	100	199·5	297·9	394·8	489·7	582·0
674·7	½	100	199·5	297·8	394·5	489·1	580·9
655·4	¾	100	199·4	297·7	394·2	488·4	579·8
637·3	9°	100	199·4	297·5	393·9	487·7	578·6
620·1	¼	100	199·4	297·4	393·5	487·1	577·4
603·8	½	100	199·3	297·3	393·2	486·4	576·2
588·4	¾	100	199·3	297·1	392·8	485 6	575·0
573·7	10°	100	199·2	297·0	392·4	484·9	573·7

For radii less than 573·7 feet, it is never required to use longer chords than 100 feet.

Remark.—Intermediate ones may be found by simple proportion.

CHAPTER III.

COMPOUND AND REVERSE CURVES, ETC.

ARTICLE XIX.

We have hitherto spoken only of *simple* curves; that is, of such as are parts of only one circle; and hence have but one radius, and *equal apex distances.*

Compound Curves Defined.

When a curve, as $a\,p\,z$, Fig. 16, has apex distances, $x\,a$, $x\,z$, of different lengths, it must also have at least two different lengths $p\,c$, $p\,o$, of radius; and if the curve also runs in one general direction, like $a\,p\,z$, (instead of in two directions, like $a\,r\,w$, Fig. 18), it is called a compound curve.

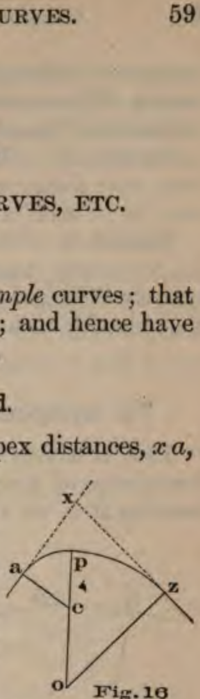


Fig. 16

The point p, at which the change of radius occurs, is called the **Point of Compound Curvature**; and the stake at that point in a survey, is marked P C C.

Reverse Curves Defined.

When one of two adjacent tangents ($x\,y$ and $y\,z$, Fig. 17; or $x\,y$ and $y\,w$, Fig. 18) deflects to the right, and the other to the left, their two curves $a\,p$ and $q\,z$, Fig. 17, must also deflect in different directions, as seen in the two Figs. When these curves, as $a\,r$ and $r\,w$, Fig. 18, *touch* each other, as at r, Fig. 18, the two together constitute a *true* **Reverse Curve.** They are often called reverse curves, even when, as in Fig. 17, they are separated by a short tangent $p\,q$.

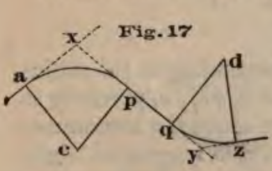


Fig. 17

Remark I.—Reverse curves should never be run without such an intermediate tangent, if it is possible to avoid doing so; because the omission prevents the proper elevation of the outer rail of the curves, and thus enforces a reduction of speed in travelling around them.

Remark 2.—The two branches or arcs of a reverse curve may have either equal or unequal radii; or each branch may be in itself a compound curve.

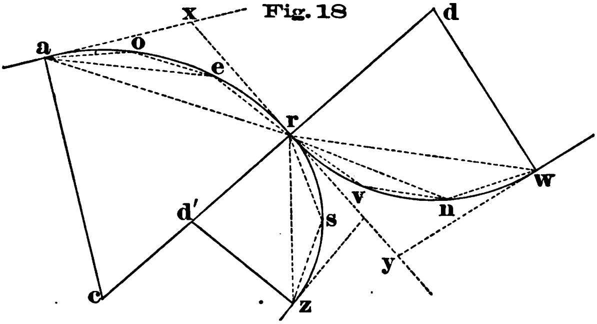
Remark 3.—The introduction of the tangent *p q*, Fig. 17, evidently transforms a true *reverse* curve into two entirely detached *simple* ones, *a p*, *q z*, each subject to all the rules for such.

ARTICLE XX.

The laying-out of Compound and Reverse Curves.

This is a very simple operation, requiring no further knowledge of principles than is taught in Art. I. Thus, starting at *a*, we run the first branch *a r* by means of tangential angles *x a o*, *o a e*, *e a r*, corresponding to the radius *a c* or *r c;* and with 100 ft. chords *a o*, *o e*, *e r*, precisely as in Art. I. Arriving at the end *r*, we move the instrument to that point, and lay off either a single tangential angle *e r x*, or the triple one *a r x*, etc., thus bringing the telescope to sight along *x r*. Revolving the telescope, it will sight along the tangent *r y*, or *x r* continued. Now it is plain that if, starting from this tangent *r y*, we continue to lay off the same tangential angles as before, we shall thereby extend the curve *a r* of radius *r c*. But if we lay off new tangential angles *y r s*, *s r z*, etc., corresponding to the radius

Fig. 18



r d′ or *z d′*, we shall complete the compound curve *a r z.* Or if, from the *opposite side* of the same tangent *r y*, we lay off tangential angles *y r v*, *v r n*, *n r w*, etc., corresponding to the radius *r d* or *w d*, we thereby complete the reverse curve *a r w.*

ARTICLE XXI.

Having given the two unequal apex distances a p, p z, *Figs.* 19 *and* 20; *and the total deflection angle, or total central angle,* v + w, **Required** *to find two radii* a c *and* z d, *for a compound curve a b z, to unite a and z tangentially.*[*]

RULE.

1st. Find the sum, and also the difference, of the apex distances.

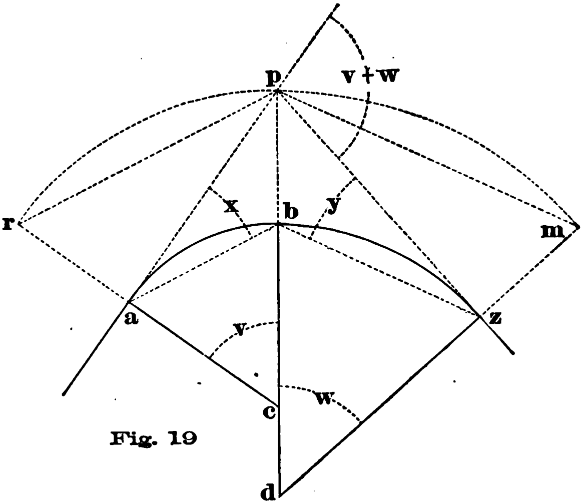


Fig. 19

Remark.—If, as in Fig. 21, the tangents are *parallel*, there will be no apex distances. For such cases see Art. XXII.

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[*] The above rule enables us to join any two points. It always gives a curve in which the line *c b*, dividing the two branches, will, when extended, strike the apex *p*; which will rarely happen when the radii are determined upon beforehand, as in Art. XXIII.

2d. Divide the total deflection angle, $v + w$, by 2. The quotient will be the *sum* of the two tangent angles x and y, Fig. 19. Call this sum s; and call their unknown difference d.

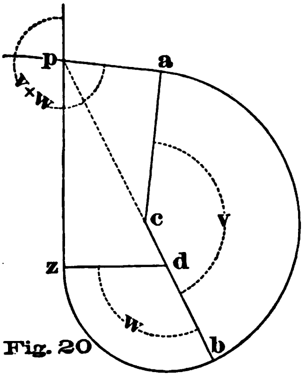


Fig. 20

Remark.—The same principle applies to Fig. 20, but cannot be conveniently illustrated in that Fig. Many of the dotted lines in Fig. 19 are merely to aid in our subsequent demonstration.

3d. Then say, as the

Sum of the apex dists. : Diff. of the apex dists. :: Sine of s : Sine of d.

4th. From the table of sines take the angle d corresponding to the sine last found.

5th. Add together s and d. Divide their sum by 2. The quotient will be x, the greater of the two angles; and $s - x$ gives y, the lesser angle.

6th. Find the angle $v =$ twice x; and the angle $w =$ twice y.

7th. Radius dz is $= \dfrac{\textit{apex distance } p\,z}{\textit{tangent of } w}$;

and Radius ca is $= \dfrac{\textit{apex distance } p\,a}{\textit{tangent of } v}$.

Remark.—When this rule is used the *larger* angle, v, always corresponds to the *shorter* apex distance, $a\,p$.

DEMONSTRATION.

The foregoing process is based upon the principle that, as the

Sum of the tangents of any two angles : Diff. between said two tangents :: Sine of the sum of the two angles : Sine of the diff. of the two angles.

Referring to Fig. 19, we have

$ra = pa \times$ tang. of rpa. Also $rpa = x$. $\therefore ra = pa \times$ tang. of x.

Also $pb = ra$. $\therefore pb = pa \times$ tang. of x.

Also $mz = pz \times$ tang. of mpz. And $mpz = y$. $\therefore mz = pz \times$ tang. of y.

Also $pb = mz$. $\therefore pb = pz \times$ tang. of y.

$\therefore pa \times$ tang. of $x = pz \times$ tang. of y; or, as $pa : pz$ :: tang. of y : tang. of x.

pa, then, represents tang. of y; and pz represents tang. of x; and we have, as in 3d,

$pa + pz : pa - pz$, or $pz - pa$ :: Sine of $(x + y)$: Sine of $(x - y)$.

ARTICLE XXII.

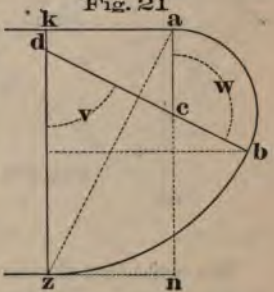
To connect tangentially by a compound curve, a b z, *Fig.* 21, *two points* a *and* z, *not* directly opposite each other, *on* **parallel** tangents *k a* and *z n*.

Required to find the radii *c b* and *d b*.

We must first find the central angles, *v* and *w*, of the two branches of the curve, thus:

Rule.—Divide the perpendicular distance *k z* between the tangents, by the distance *k a* which the ends of the curve want of being opposite each other. The quotient will be the tangent of *k a z*. Now *k a* is perpendicular to *d z*, and by drawing *d b* perpendicular to *a z* we make *z d b*, or *v*, equal to *k a z*.* Therefore $kz \div ka =$ tangent of *v*.

Fig. 21



Find *v* in the table of tangents; and deduct it from 180°. The remainder will be the angle *w*; because when tangents are parallel, $v + w$ must always be 180°.

From the table take the versed sines of *v*, and of *w*. Then

---

* The points *a* and *z* may be joined by other compound curves, in which *d b* is *not* perpendicular to *a z*, but by making it perpendicular we obtain a satisfactory curve, and a simple method of finding the radii.

Radius cb = *half* kz ÷ versed sine of w; and
Radius db = *half* kz ÷ versed sine of v;
for when the tangents ka and zn are parallel, a line an joining them at right angles is $= (db \times \text{ver. sine of } v) + (ac \times \text{ver. sine of } w)$.

And when db is made perpendicular to az, then $(ac \times \text{ver. sine of } w) = (db \times \text{ver. sine of } v) = \text{half } an$.

ARTICLE XXIII.

Having the total deflection angle, x′, *Figs.* 22 *and* 23, *between two tangents,* a c *and* c b, *which are to be united by a compound curve* a p b, *starting from* a, *it is*

Required, *to find how far* a c, *from the apex,* c, *to begin the curve.* See footnote to Art. XXVI.

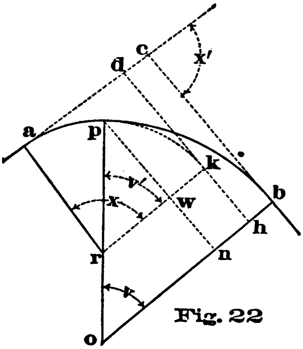


Fig. 22

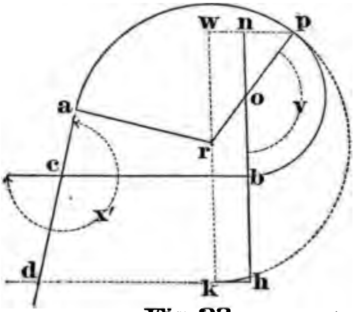


Fig. 23

We must first decide what radii, pr and po; and what lengths, in chords, the two branches, ap and pb, of the curve shall have.

Rule.—Find the central angle, v, subtended by the *second* branch, pb, of the curve, = its chord-deflection angle × its number of chords.

Find, in the table, p. 170, the versed sine of this angle, v.

Find the difference, ro, in length between the radii po and pr.

Find $b h = r o \times$ versed sine of v.*

Find in the table, p. 124, the sine of x'.

Find $d c = b h \div$ sine just found.

Find in the table, p. 124, the tangent of *half* the central angle x.

Find $a d =$ radius $a r \times$ tangent just found.

Find $a c$, thus:

Case I.—When x is less than $180°$, $a c = a d + d c$.

Case 2.—When x exceeds $180°$, and the *shorter* radius is run first, $a c = a d + d c$.

Case 3.—When x exceeds $180°$, and the *longer* radius is run first, $a c = a d - d c$, or

Case 4. $a c = d c - a d$.

For the starting point a, from c measure $a c$ *backward*, in Cases 1 and 4; or *forward* in Cases 2 and 3.

ARTICLE XXIV.

Having the total deflection angles x b i *and* w c i, *Fig.* 24, *and the distance* b c, *it is,*

Required *to find the greatest radius,* g i *or* h i, *that can be employed in a reverse curve,* f o i n m, *for uniting* a b *to* c d; *and to locate the point of curve,* f; *or point of tangent,* m.

Rule.—From the table, p. 124, take the tangent of $b g i$, $=$ *half* of $x b i$; and the tangent of $i h c =$ *half of* $w c i$. Add these two tangents together, and divide $b c$ by their sum. The quotient is the greatest common radius, $g i$ or $h i$.

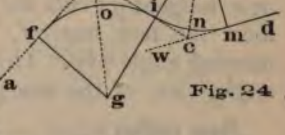


Fig. 24

Find the apex distance $b i$ or $b f =$ radius $g i \times$ tangent of $b g i$.

Find the apex distance $c i$ or $c m =$ radius $h i \times$ tangent of $i h c$; or $= b c - b i$.

Example.—Let $x b i$ be $108° 20'$; $w c i$ $50° 45'$; and the distance $b c$ 950 feet. What is the length of radius $h i$ or $g i$, of the easiest reverse curve for uniting $a b$ to

---

* Because if we imagine the first branch, $a p$, of the curve to be extended forward to k, so as to subtend the angle $x = x'$, and from p draw $p w n$ at right angles to $o b$, then $w k$ $(= n h)$ is $= r p \times$ ver. sine of v' $(= v)$; $n b = o p \times$ ver. sine of v; and $b h$ is $= n b - n h$, or $n h - n b$ $=$ the difference $r o$ of the radii $\times$ ver. sine of v.

$c\,d$; how far, $b\,f$, back from b must the curve begin; and how far, $c\,m$, forward from c will it end?

Here we have,

half of $x\,b\,i$ (108° 20′) = 54° 10′; its tangent 1·3848;
and half of $w\,c\,i$ (50° 45′) = 25° 22½′; its tangent ·4743.

The sum of these tangents = 1·3848 + ·4743 = 1·8591.

Radius $g\,i$ or $h\,i = b\,c$ (950 ft.) ÷ 1·8591 = 511 ft.

Apex distance $b\,i$ or $b\,f$ = Rad. $g\,i$ or $h\,i$ (511 ft.) × tang. of *half* $x\,b\,i$ (1·3848) = 707·63 ft.

Apex distance $c\,i$ or $c\,m$ = Rad. $g\,i$ or $h\,i$ (511 ft.) × tang. of *half* $w\,c\,i$ (·4743) = 242·37 ft.; or $= b\,c - b\,i =$ 242·37 ft.

ARTICLE XXV.

To alter the last part of a Curve so that it will properly join a New Tangent.

Remark.—This problem covers most of the cases that occur in practice; but at times certain restrictions present themselves which require other methods, some of which will be found further on.

Having from a, *Figs.* 25, 26, 27, *and* 28, *run a curve* a b, *ending in a tangent* b e, *we wish to alter the radius of the last part of it, so that it shall connect tangentially with a new tangent* g f, *either parallel to the old one* b e, *or not.*

Rule.—From any point o of the curve (which point must be back *from s*, when, as in Figs. 25 and 26, the new tangent *$g\,f$ cuts the curve*) run a short tangent $o\,t$ to meet the new one $g\,f$. Measure both $o\,t$ and the outer meeting angle $o\,t\,g$. This angle will be equal to the central angle, $o\,x\,n$, of the new part, $o\,n$, of the curve. Hence

$$\textbf{New radius } o\,x = \frac{o\,t}{\text{tang, of } \tfrac{1}{2}\, o\,x\,n} = \frac{o\,t}{\text{tang. of } \tfrac{1}{2}\, o\,t\,g}.$$

Remark 1.—If we do the same thing at a, we shall get a radius which will give a *uniform* curve from a to the new tangent $g\,f$; and this will often be better than the compound curve otherwise obtained.

Remark 2.—When, as in Figs. 25 and 26, the new tangent *$g\,f$ cuts the curve*, as at s (or if it should cut an *extension* of the curve forward from b), then any new radius $o\,x$ will be *shorter* than the old one $b\,c$; lengthening, however, the farther we go back from *b;* but when, as in Figs. 27 and 28, the new tangent does *not* cut the curve (or an extension of it forward from *b*), any new radius *o x* will be *longer* than the old one *b c;* shortening, however, the farther we go back from *b.*

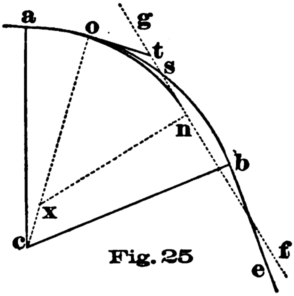


Fig. 25

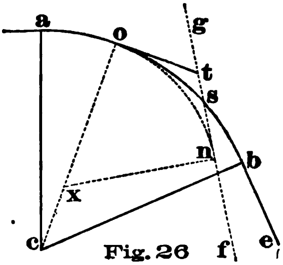


Fig. 26

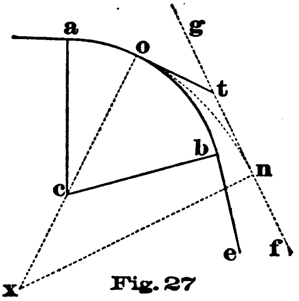


Fig. 27

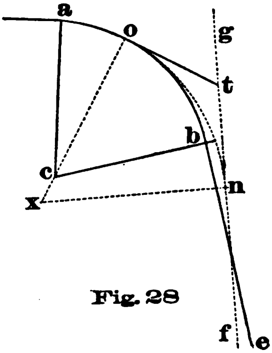


Fig. 28

Remark 3.—It does not always follow that the end of the new part of the curve of greater radius than the old part will always extend *beyond b,* as in Fig. 27; nor that one of shorter radius will fall *behind b.*

Remark 4.—In curves not exceeding 180° long, if the error is small, **it may be humored in** by dividing it equally among the chords by measure, without retracing the curve with an instrument. This method may be employed with perfect security so long as the error does not exceed 1 foot to every chord of 100 feet; and it will never be so great if moderate care be taken.

Thus, if the curve be 20 chords long, and the error 20 feet, the last stake may be moved 20 feet, the next 19, the next 18, etc., as nearly at right angles to the curve as can be judged by the eye.

The same ordinates that would have been used had the curve been correct, will answer for the one so adjusted, without perceptible difference.

The resulting curve will not be truly circular, although very closely so; still, for the sake of uniformity throughout the route, it had as well be corrected at leisure, before actual grading begins, if the error exceeds 2 or 3 inches per 100 ft. chord.

ARTICLE XXVI.

Having from **a**, *Figs.* 29 *and* 30, *run a curve* **a b** *to the tangent* **b s**, *it is desired,* **with the same radius,** *to strike the* **parallel tangent** *o g*, either inside of *b s*, as in the Figs., or outside of it; *the perpendicular distance,* b c, *between the tangents being given.*

Required, *the distance,* a n, *for starting the new curve* n o.*

Rule.—In either Fig. find the sine of the central angle

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*It must be **carefully** borne in mind that in Fig. 30, as well as in all that follow in which **the curve is greater than 180°**, the **central** angle is the **larger** one at the centre, and is the one there **subtended by the curve.** The smaller one at the same centre, and denoted by the very same letters, is what we have called the **substitute** angle. See page 30. In some of the problems this is substituted for the true central angle; but when so, the fact is stated. No error, however, would arise from taking the sine, cosine, tangent, etc., of one for that of the other, inasmuch as they are the same for both angles.

In the simple curve *a e b*, Fig. 30, the true central angle *a x b* is that on the right of the two radii *a x* and *b x*; while the angle *a x b* on the left of those radii is the *substitute* angle; always equal to what the central angle wants of being 360°.

The same care is necessary with regard to the total deflection angle, (shown by the dotted arc around *r*, Fig. 30) where two tangents of a curve of more than 180° meet. Whether the curve is greater or less than 180°, this angle, in a *simple* curve, is always equal to the central angle; or, in a *compound* curve, to the sum of its two or more central angles.

axb, subtended by the curve. Divide the given distance bc by this sine. The quotient will be the distance $bo = an$; to be measured *backward* from a to n, when, as in Fig. 29, the new tangent og is back towards the beginning of the survey, from the old tangent bs; and to be measured *forward* from a to n, when, as in Fig. 30, the new tangent og is forward towards the end of the survey, from the old tangent bs.

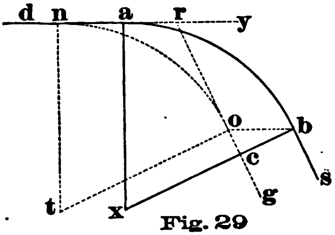

Fig. 29

Remark.—If the central angle axb is just 90° (as it would be with the tangents ek and wm in Fig. 30); or just 270°, the sine is 1; and the dist an is $= ew \div 1 = ew$. If the central angle

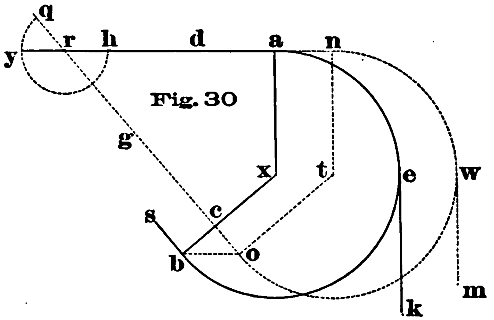

Fig. 30

be 180°, the sine is 0; and the problem is impossible. In this case a change of tangents would require a change in the length of the radius.

DEMONSTRATION.

Since the two curves ab and no, in either one of the figs., starting from the same tangent nr, are equal, bo must be equal to its parallel an. And as bo is made parallel to the tangent, da, boc is, in Fig. 29, evidently equal to total deflection angle $yro = axb$; and in Fig. 30, to the supplement, yrq, of total deflection angle drq.

Now an angle and its supplement (see p. 121) have the same sine.

As bx is perpendicular to og, bc is $= bo \times$ sine of boc or of axb.

In other words,

$$an\ (= bo)\ \text{is} = \frac{bc}{\text{sine of } boc} = \frac{bc}{\text{sine of central angle.}}$$

ARTICLE XXVII.

Having from a *run a curve,* a c, *Figs.* 31 *and* 32, *ending in a tangent,* c n, *it is desired to start again at the* **same point,** a, *and to run a curve,* a b, *either of longer radius, as in the figs., or of shorter radius, to unite with a tangent, as* b m, *parallel to* c n; *and either outside of* c n, *as in the figs., or inside of it.*

Required, the new radius. See footnote to Art. XXVI.

Remark.—*This rule applies equally to the case of a compound curve, as* w a c, *where it is desired to retain the same point,* a, *of compound curvature.*

Here we first find the *difference* between the two radii; and then either add it to, or take it from, the first radius to get the new one.

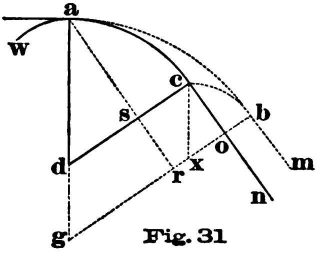

Fig. 31

Rule.—Measure the shortest distance, ob, between the tangents, bm and cn.

Divide this ob by the versed sine of the central angle, adc, subtended by the curve. The quotient is the difference, dg, between the radii.

Then if the new tangent, bm, lies *outside* the old one, cn, as in the figs., *add* dg to the old radius, ad, for the new one, ag.

If the new tangent lies *inside* the old one, *subtract* dg from the old radius, for the new one.

Remark.—If the central angle is just 90°, as ade, Fig. 32, or just 270°, the difference dg between the radii will be equal to the distance fh between tangents fi and hj; because the versed sine of 90°, or of 270°, is 1.

If the total angle is just 180°, as agq, Fig. 32, the difference dg between the radii will be equal to *half* the distance pq between the tangents pv and qu; because the versed sine of 180° is 2.

DEMONSTRATION.

From a draw the dotted line ar, parallel to the two tangents, bm and cn, and therefore perpendicular to dc and gb. Then $sc =$ radius $ad \times$ versed sine of central angle; and $rb =$ radius $ag \times$ versed sine of central angle.

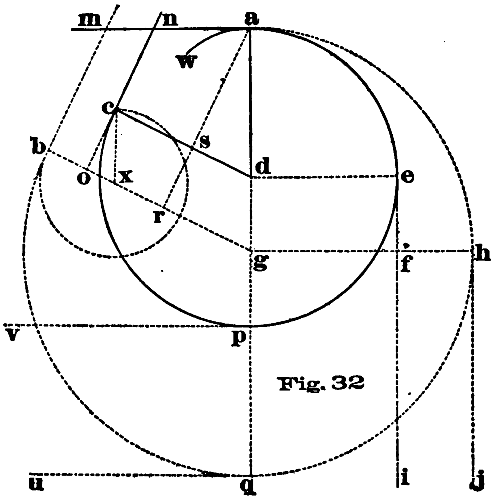

Fig. 32

The distance bo between the tangents is $= rb - sc =$ ($ag \times$ ver. sine of central angle) — ($ad \times$ ver. sine of central angle) $= (ag - ad) \times$ ver. sine of central angle, as shown by the dotted line, xc, and dotted arc, cb.

Hence difference dg of radii, or $ag - ad$, is

$$= \frac{bo}{\text{ver. sine of central angle}}.$$

ARTICLE XXVIII.

Having from a, *Figs.* 33 *and* 34, *run a curve*, a b, *ending tangentially in* b c, *we wish to* **change the starting-point** *from* a *to* f, *so that* **with the same radius,** e a, *we shall strike, as at* i, *a new tangent*, b d, *passing through* b, *and deflecting either inward or outward from* b c.

Required *the distance* a f. See footnote to Art. XXVI.

RULE.

1st. Place the instrument at b, and measure the angle dbc.

2d. Find the new central angle, fgi ($= und$), thus: If, as in Fig. 33, the new tangent, bd, deflects *outward*, then fgi is = old central angle aeb (subtended by the curve) *minus* dbc. Or if, as in Fig. 34, the new tangent, bd, deflects *inward*, then fgi (of the entire dotted arc fi) is = old central angle, aeb (of the entire arc ab), *plus* dbc.

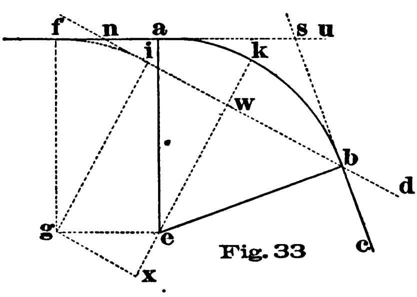

Fig. 33

Remark.—If the new tangent, bd, is *parallel to the original one*, a s (in other words, if fgi is 180°), then *the radius must be shortened* so as to be = *half* the perpendicular distance between as and the new tangent, and the curve may start from any convenient point, as a, f, etc., etc., on the tangent nu.

3d. Find ex ($= kw$) = radius $ea \times$ ver. sine of dbc.

4th. Find ge ($= af$) = $ex \div$ sine of fgi.

5th. If, as in Fig. 33, the new central angle is *less* than 180°, measure af, thus found, *backward* from a. If, as in Fig. 34, the new central angle *exceeds* 180°, measure af *forward* from a.

Example.—In Fig. 34 the new tangent, bd, deflects *inward* from the first one, bc. Therefore,

1st. Measure $d\,b\,c$ = say 40°.

2d. Find $f\,g\,i$ or $u\,n\,d = a\,e\,b$ (say 250°) + $d\,b\,c$ (say 40°) = 290°.

3d. Find $e\,x$ (= $k\,w$) = radius $e\,a$ (say 100 ft.) × ver. sine (·234) of $d\,b\,c$ = 23·4 ft.

4th. Find $a\,f = e\,x$ (23·4) ÷ sine (·940) of $f\,g\,i$ = 24·9 ft.

5th. Since $f\,g\,i$ *exceeds* 180°, measure $a\,f$ (24·9 ft.) *forward* from a for the new starting-point f.

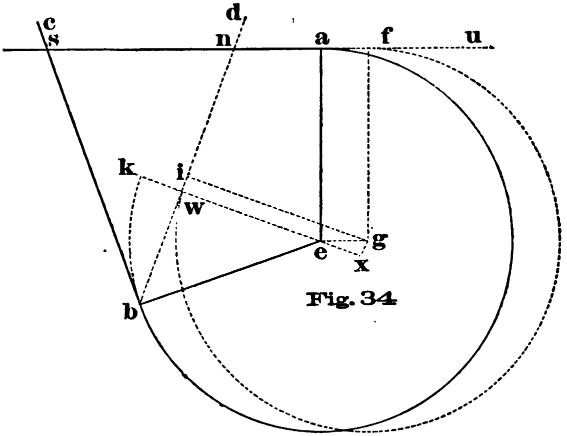

Fig. 34.

DEMONSTRATION.

Draw the line $k\,w\,e\,x$, Figs. 33 and 34, perpendicular to new tangent $b\,d$, and passing through the old center e. Since $k\,e$ and $b\,e$ are respectively perpendicular to both the new and the old tangents, $b\,e\,k$ is = $d\,b\,c$. If the curve were to be pushed back from k, along the line $k\,x$, it evidently must, in order to become tangential to $b\,d$, move through the distance $e\,x$ (= $k\,w$) = radius × ver. sine of $k\,e\,b$ or of $d\,b\,c$. But the curve must be tangential, not only to $b\,d$, but also to $n\,u$; and the center of the curve must, therefore (after moving from e to x), move along the line $x\,g$, perpendicular to $e\,x$, and parallel to $b\,d$, until its perpendicular distance $f\,g$ from $a\,s$ is = radius $e\,a$.

Now, to find the distance $e\,g$ (= $a\,f$), through which to move the curve at once from e or a, we have, Since $x\,g$ and $e\,g$ are respectively parallel to $b\,d$ and $n\,u$, $e\,g\,x$ is $= u\,n\,d = f\,g\,i$. In the right-angled triangle $e\,x\,g$, $e\,x$ is evidently $= e\,g \times$ sine of $e\,g\,x$ (or of $f\,g\,i$). In other words, $e\,g$ ($= a\,f$) is $= e\,x \div$ sine of $f\,g\,i$.

ARTICLE XXIX.

Having from a, *Figs.* 35 *and* 36, *run a curve*, a b, *to* b, *where it is tangential to* b c, *we wish to find* **a new radius,** a g, *for a new curve*, a i, *to begin* **at the same point,** a, *and end, as at* i, *in the new tangent*, i d, *which intersects the old tangent*, b c, *at* b.

Required the new radius $a\,g$ or $i\,g$. See footnote to Art. XXVI.

Remark I.—In this problem, the new radius will always be shorter than the old one.

Remark 2.—If the new tangent, or its extension backward, cuts the old radius, $a\,e$, as in Fig. 33, this problem does not apply. In such cases, the starting-point, a, must be moved. See Arts. XXVIII. and XXX.

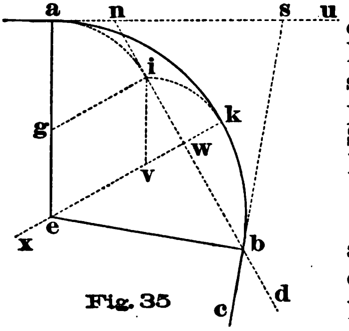

Fig. 35

RULE.

1st. Place the instrument at b, and measure the angle $d\,b\,c$ formed by the old and the new tangents.

2d. Find the new central angle $a\,g\,i$ (subtended by the entire new curve $a\,i$) $= a\,e\,b$ **minus** $d\,b\,c$, when, as in Fig. 35, the new tangent, $b\,d$, deflects *outward* at b; or $= a\,e\,b$ **plus** $d\,b\,c$, when, as in Fig. 36, it deflects *inward.*

3d. Find $e\,x$ ($= k\,w$) $=$ radius $a\,e \times$ ver. sine of $d\,b\,c$.

4th. Find $g\,e$, the difference between the old and the new radius $= e\,x \div$ ver. sine of $a\,g\,i$ or of $u\,n\,d$.

5th. Find the new radius $a\,g =$ old radius $ae - g\,e$.

DEMONSTRATION.

To show that ex is = radius ae × versed sine of dbc, see demonstration of Art. XXVIII.

To show that ge is = ex ÷ versed sine of agi; from i draw iv, parallel and equal to ge. Now, iv and kv being respectively parallel to ag and ig, the arc ik is = arc ai, and the angle ivk is = angle agi. Also, kw (= ex) is evidently = iv × versed sine of ivk (= ge × versed sine of agi). In other words, ge is = ex ÷ versed sine of agi.

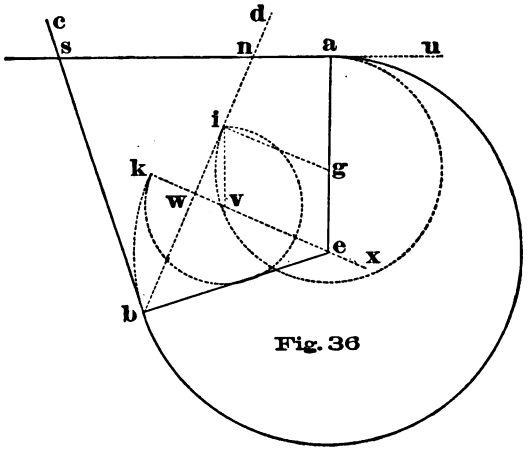

Fig. 36

ARTICLE XXX.

Having from a, *Figs.* 37 *and* 38, *run a curve,* a b, *to* b, *where it is tangential to* b c, *we wish to* **change the radius and the starting-point,** *so as to strike the divergent tangent,* b d, **at the same point** b.

Required the new radius fg and the distance af. See footnote to Art. XXVI.

Rule.—For the new radius fg.

1st. Place the instrument at b, and measure the angle dbc.

2d. Find the new central angle fgb, subtended by the entire new curve fb, *thus:* If, as in Fig. 37, the new tangent, *b d*, deflects *outward*, then *f g b* (= *u n d*) is = old central angle *a e b* **minus** *d b c*. But if, as in Fig. 38, the new tangent, *b d*, deflects *inward*, then *f g b* =old central angle *a e b* **plus** *d b c*.

3d. Find *f k* (= *a w*) = old radius *a e* × ver. sine of *a e b*.

4th. Then new radius *f g* = *f k* ÷ ver. sine of *f g b*.

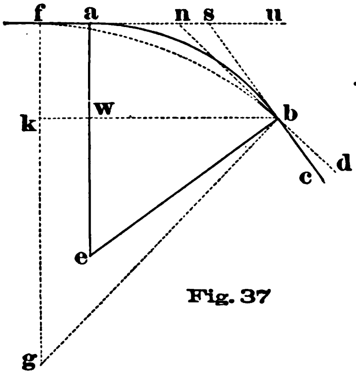

Fig. 37

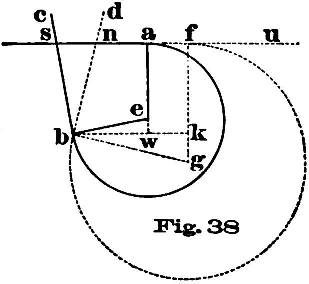

Fig. 38

Remark.—Having now the new radius, it is plain that nothing more is necessary than to run the new curve, *f b*, *backward*, beginning at *b*. This will, of course, bring us to *f*, without any special calculation for finding *a f*. It may, however, be found thus:

1st. Find *b k* (= new radius *f g* × sine of *f g b*).

2d. Find *b w* (= old radius *a e* × sine of *a e b*).

3d. Find *a f*, thus:

If *a e b* and *f g b* are both greater (as in Fig. 38), or both less (Fig. 37), than 180°, then

a f is = the **difference**, *k w*, between *b k* and *b w*.

If either *a e b* or *f g b* is greater, and the other less, than 180°, then

a f is = the **sum** of *b k* and *b w*.

4th. From *a*, measure *a f* along *s a*; *toward b* if the new radius is *less* than the old; and *away from b*, if, as in Figs. 37 and 38, the new radius is *greater* than the old.

ARTICLE XXXI.

Having run a curve from o *to* a, *Figs.* 39 *and* 40, *and then a tangent to* b, *we wish to find a new point,* e, *on the curve, such that a new tangent,* e c, *shall pass through a point,* c, *at a given distance,* b c, *at right angles to* a b.

In Fig. 39, the point c is *outside* the tangent $a\,b$; and in Fig. 40 it is *inside.*

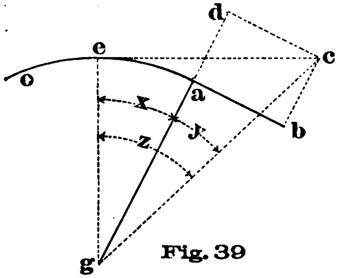

Fig. 39

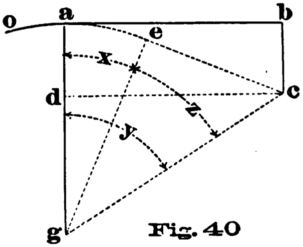

Fig. 40

Required, to find the angle x, which is the number of degrees to go *backward* along the curve from a to e, in Fig. 39; or *forward* in Fig. 40.

Rule.—Complete the rectangle, $a\,b\,c\,d$, thus making $d\,c = a\,b$; and $a\,d = b\,c$; and thus finding $g\,d$, = the **sum** of $g\,a$ and $a\,d$ in Fig. 39; and = their **difference** in Fig. 40.

Find the angle y, by means of its tangent $\frac{d\,c}{g\,d}$. Find its sine.

Find $c\,g$, $= d\,c \div$ sine of y.

Find the angle z, by means of its cosine $\frac{e\,g}{c\,g}$.

Find the angle x, = the *difference* between y and z.

This x is the number of *degrees* from a to e; and consequently $\frac{x}{\text{chord-deflection angle}}$ is = the number of *chords* from a to e.

ARTICLE XXXII.

Having from a *laid out a simple curve to* b, *Figs.* 41 *and* 42, *and at* b *changed it to a compound curve by adding* b c, *so that at* c *it terminates in the tangent* c n; *it is*

Required *to find a new point,* k, *of compound curvature, so that the compound curve,* a k f, *traced* **with the same radii** as before, *may terminate, as at* f, *in a new tangent,* f m, **parallel** *to the first one,* c n, *and either outside or inside of it.*

There are two cases:

Case I.—A. (Fig. 41) When the curve of *shorter* radius is run first, and the tangent sought is *inside* the first one.

B. (Fig. 42) When the curve of *longer* radius is run first, and the tangent sought is *outside* of the first one.

Case 2.—C. When the curve of *shorter* radius is run first, and the tangent sought is *outside* of the first one.

D. When the curve of *longer* radius is run first, and the tangent sought is *inside* of the first one.

If we suppose the first branch, *a b*, of the curve, to be extended forward to a point, *t*, where it would be tangential to *j v*, parallel to the two tangents, *c n* and *f m;* then:

Case 1, as in our figs., is when *j v* and *f m* are on the *same* side of *c n;* and

Case 2 is when *j v* and *f m* are on *opposite* sides of *c n*.

Remark.—In Case 1, the tangent *f m* sought, must lie between the first tangent *c n* and the tangent *j v* to the first branch extended. Otherwise this problem does not apply, and the radii must be changed.

Both our figures illustrate Case 1, but by changing the lettering they can be made to answer for Case 2.

In both figures we have denoted equal angles by the same letter, without reference to their position in the figure.

Rule, *both in* Case 1 and in Case 2.

1st. *From the* Table, p. 170, take the versed sine of the central angle, *g*, subtended by the second branch, *b c*, as first run.

2d. Measure on the ground the distance, *r s*, at right

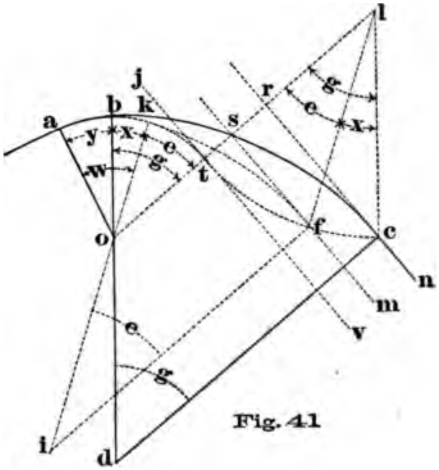

Fig. 41

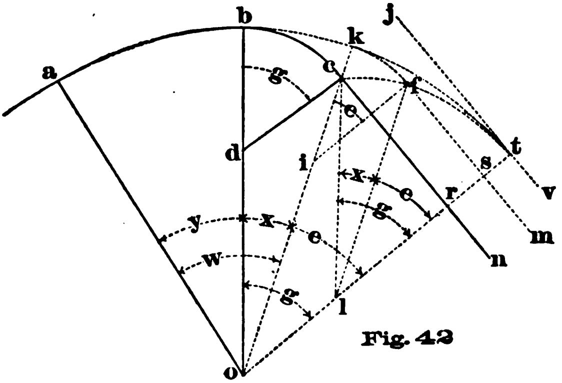

Fig. 42

angles between the tangents *c n* and *f m*. Divide *r s* by the difference, *d o*, between the radii. Call the quotient $\frac{r\,s}{d\,o}$.

3d. Find the new central angle, *e*, for the second branch, *k f*, by means of its versed sine, which is equal

in Case 1 to the versed sine of *g* **minus** $\frac{r\,s}{d\,o}$; and

in Case 2 to the versed sine of *g* **plus** $\frac{r\,s}{d\,o}$.

Remark.—If, in Case 2, this sum exceeds 2, either the radii or the starting-point, *a*, must be changed, and this problem does not apply.

Remark.—Either of the two angles given by the table for a versed sine, found as above, may be taken as *e;* but the one nearest equal to *g* will give the shortest distance to run backward or forward from *b*.

4th. Now, as the tangents *c n* and *f m* are parallel, the *total* central angle will be the same after the change as before it. In other words $(w + e)$ will be equal to $(y + g)$. In order to bring this about, we must *increase y* just as much as we *diminish g,* and *vice versa.*

Therefore, find *x*, which is equal to the difference between *g* and *e*, find the number of chords in *b k*, which is $= \frac{x}{\text{chord-defl. angle of first branch } a\,b}$; and from *b* lay off this number of chords, *forward* if *g* is greater than *e*, *backward* if *e* is greater than *g*.

Remark.—If, in going *backward* from *b*, we find that *x* exceeds *y*, we must change the radii or the point *a*.

Example.—CASE 1. Suppose a 2° curve (radius 2865 feet) followed by a 3° one (radius 1910 feet), the second branch having a central angle of 36°. It is required to strike a tangent 20 feet *outside* the present one.

Here, versed sine of 36° *minus* $\frac{20 \text{ ft.}}{\text{diff. of radii (955)}} =$ versed sine of new central angle.

Or, ·1910 *minus* ·0209 = ·1701 = versed sine of 33° 54′.

And 36° — 33° 54′ is = 2° 6′, which is = $1\frac{5}{100}$ chords of a 2° curve. Therefore go *forward* $1\frac{5}{100}$ chords from *b*, with first radius *a o*, to *k*.

CASE 2. Suppose a 2° curve followed by a 3° one;

central angle of second branch 36°. It is required to strike a tangent 20 feet *inside* the present one.

Here, versed sine of $e =$ versed sine of g *plus* $\frac{rs}{do} = .1910 + \frac{20}{955} = .1910 + .0209 =$ versed sine of 38°.

And $g - e = 38° - 36° = 2° =$ 1 chord of a 2° curve.

Therefore, go *back* 1 chord from *b* with first radius, *a o*, to *k*.

DEMONSTRATION

of Case 1, by Figs. 41 and 42.

From *c*, where the *old* second branch, *b c*, joins its tangent, *c n*, draw *c l* parallel to *d o*, the difference between the radii, and evidently equal to it; thus making *c l t* equal to *g*. As *c r* is perpendicular to *o t*, *r t* is $= d\,o \times$ versed sine of *g*.

From *f*, where the *new* second branch, *k f*, joins its tangent, *f m*, draw *f l* parallel and equal to *o i*, and equal to *d o*. Then will *f l t* be $= e$; and *s t* will be $= d\,o \times$ versed sine of *e*. Now *s t* is $= r\,t - r\,s$: in other words, ($d\,o \times$ versed sine of *e*) is $= (d\,o \times$ versed sine of $g) - r\,s$.

Hence we have, as in "3d" of the rule,

$$\text{Versed sine of } e = \text{versed sine of } g - \frac{rs}{do}.$$

ARTICLE XXXIII.

Having from b *or* d *run a curve,* b x o, *or* d y t, *Figs.* 43 *and* 44, *of known radius, and central angle,* b c o *or* d n t, *we wish to* **change the radius,** *so that the new curve shall, at its middle point,* x *or* y, *be a given distance,* x y, *from the old one.*

Required the new radius and *b d.*

Case 1, Fig. 43. **When the central angle is less than 180°.**

A.—*When the* **outer** *curve,* b x o, *has been run first.*

RULE.

1st. Find $a\,e =$ radius $b\,e \div$ cosine of $b\,e\,a =$ radius $b\,e \div$ cosine of *half* *b e o*.

2d. Find $ax = ac$ — rad. bc.
3d. Find $ay = ax +$ given distance xy.
4th. Find new radius, dn, thus:
$ax : ay ::$ old rad. bc : new rad. dn.*
5th. Find $bd = (dn - bc) \times$ tangent of bca.†

Example.—Let rad., bc, of the curve, bxo, first run, be 300 ft.; the central angle, $bco = 100°$; and the given distance, $xy = 80$ ft.

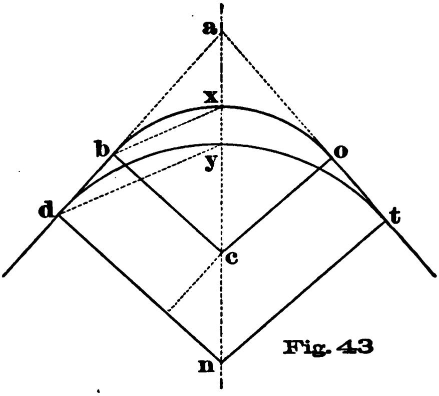

Fig. 43

Then we have,

1st. $ac =$ radius $bc \div$ cosine of $bca = bc \div$ cosine $50° = 300 \div .6428 = 466.7$ ft.
2d. $ax = ac - bc = 466.7 - 300 = 166.7$ ft.
3d. $ay = ax + xy = 166.7 + 80 = 246.7$ ft.
4th. $ax : ay :: 166.7 : 246.7 :: bc : dn :: 300 : 444$. $dn = 444$.
5th. $bd = (dn - bc) \times$ tang. $bca = cn \times$ tang. $50° = (444 - 300) \times 1.19175 = 144 \times 1.19175 = 171.6$ ft.

B. *When the* **inner** *curve* d y t *has been run first.*

RULE.

1st. Find $an = dn \div$ cosine of $dna = \dfrac{dn}{\cos. \frac{1}{2}\, dnt}$.
2d. Find $ay = an - dn$.
3d. Find $ax = ay - xy$.

---

[*] Because by similar triangles, as $ax : ay :: ab : ad$; and as $ab : ad :: bc : dn$.

[†] *Because* ad is $= dn \times$ tang. of dna or bca; and $ab = bc \times$ *tang. of* bca; and $db = ad - ab = (dn - bc) \times$ tang. of bca.

4th. $ay : ax :: dn : bc$.
5th. $bd = (dn - bc) \times$ *tang.* of dna.

Case 2, Fig. 44. **When the central angle exceeds 180°.**

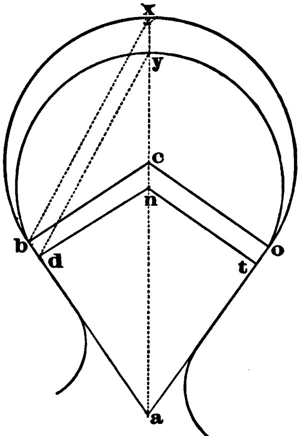

Fig. 44

Rule.—Subtract the central angle bco, or dnt (subtended by the curve), from 360°; and use the remainder (which will be our "substitute angle," or the *smaller* angle bco or dnt) as a substitute for it; proceeding precisely as in the foregoing A and B, with the following exceptions:

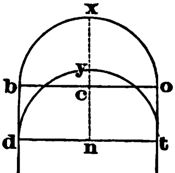

Fig. 45

(A.) When the **outer** curve, bxo, has been run first,

ax will be $= ac$ *plus* bc; and
ay will be $= ax$ *minus* xy.

(B.) When the **inner** curve, dyt, has been run first,

ay will be $= an$ *plus* dn; and
ax will be $= ay$ *plus* xy.

Remark.—Fig. 45. **When the central angle is precisely 180°;** it is evident that the radius remains unchanged, and that the dist bd is $= xy$.

ARTICLE XXXIV.

To find what radius, a c, *Figs.* 46 *and* 47, *a curve* a z, *starting at* a, *must have, and how many chords it must run; in order to pass through a point* z, *at given distances,* w z, *at right angles to the tangent* b a; *and* f z, *or* a w, *at right angles to* a f; *in front of* a, *Fig.* 46, *or behind* a, *Fig.* 47.

Rule.—Divide wz by aw. The quotient will be the tangent of x. From the table, p. 124, take x,* and multiply it by 2. The product will be the central angle t.

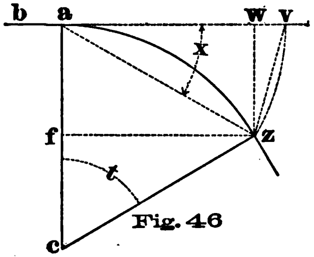

Fig. 46

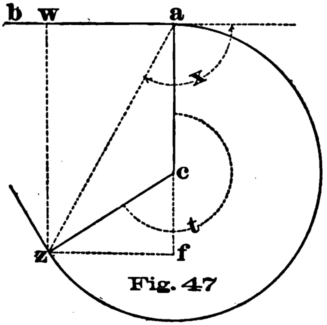

Fig. 47

Then radius ac is $= \dfrac{fz \text{ or } aw}{\text{sine of } t} = \dfrac{af \text{ or } wz}{\text{versed sine of } t}$.

And $\dfrac{\text{angle } t.}{\text{chord-def. angle}}$ = number of chords of curve from a to z.

ARTICLE XXXV.

To find how far a w, *Figs.* 46 *and* 47, *from* w *to start a curve* a z *of given radius,* a c, *in order to pass through a point* z, *at a given distance,* w z, *at right angles to the tangent* b a; *in front of* a, *Fig.* 46, *or behind* a, *Fig.* 47.

Rule.—Divide wz by the radius ac. The quotient will be the versed sine of central angle t. From the table, p. 170, take the angle t. †

Then aw = rad. $ac \times$ sine of t.

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* In Fig. 47 take the tabular x from 180°, for the *true* x.

† In Fig. 46 t is the *lesser* of the two angles in the table; in Fig. 47, the *greater*.

ARTICLE XXXVI.

To find how far, w z, *Figs.* 46 *and* 47, *a curve,* a z, *of given radius,* a c, *will be from, and at right angles to its tangent,* b a, *after running a given number of chords.*

Rule.—Multiply the chord-deflection angle by the given number of chords. The quotient will be the central angle *t*.

Find the versed sine of *t*, in table, p. 170.

Then *w z* is = radius *a c* × ver. sine of *t*.

ARTICLE XXXVII.

To Pass or Clear Obstacles.

Mode 1, Fig. 48. Suppose that in running a straight line from *a* towards *b*, we meet with an obstacle, *o*, which may be a deep pond, or a building, etc. To pass it, at any convenient point *c* in the line *a b*, lay off an angle *c* of 60°, thus sighting along *c n*. From *c* measure *c n* sufficiently long to clear the obstacle. Remove the instrument to *n*, and lay off another angle of 60°, thus sighting along *n e*. Make *n e* as long as *c n*. Then *e* will be in the straight line *a b;* and its distance from *c* will be equal to *c n* or *n e*. From *e* take sight at *n;* lay off the angle *n e c*, 60°, or the angle *n e b*, 120°, and then the telescope will sight along the line *a b*, which we are running. This is perhaps the neatest and most expeditious mode of proceeding in such cases.

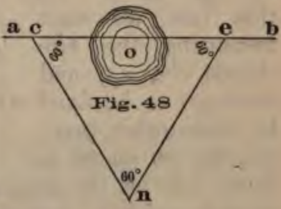

Fig. 48

Remark.—If any other angle than 60° is used at *c*, then the angle *e* must be made = *c*. The angle *n* must be made = (180°— the sum of *c* and *e*), and the distance, *c e*, will be = *c n* × 2 × cosine of *c*.

Mode 2, Fig. 49. At times this may be found more applicable than Mode 1, although a little more troublesome, as it requires four angles of 90° instead of three of 60°. Running from *m* towards *r* we have to clear the obstacle

d. At any convenient point o on the line $m\,r$, lay off the angle 90°, thus sighting towards v. Measure off any distance $o\,v$ sufficient to clear the obstacle. Move the instrument to v, and make the angle at v $90°$; thus sighting along $v\,w$. Measure $v\,w$ long enough to clear the obstacle. Move the instrument to w, and lay off the 90°, thus sighting along $w\,s$. Make $w\,s$ as long as $o\,v$. Then s will be in the straight line $m\,r$; and its distance from o will be equal to $v\,w$. Move the instrument to s, and lay off the angle $w\,s\,o$ or $w\,s\,r = 90°$; and then the telescope will sight along the line $m\,r$ which we are running.

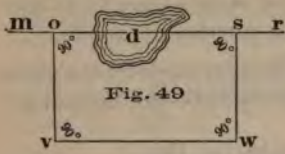

Fig. 49

Mode 3.—By long chords, Fig. 50. Having run a curve from a to d by tangential angles $o\,a\,c$, $c\,a\,d$, and with 100 ft. chords $a\,c$, $c\,d$, we find that an obstacle prevents our sighting to e. In that case we omit measuring the two chords $d\,e$, $e\,f$, and leaving e to be fixed by some other process, we go on to f, thus: Lay off the tangential angles $d\,a\,e$, $e\,a\,f$, as if no obstacle existed; and sighting along $a\,f$, make it a long chord corresponding to four stations, $a\,c$, $c\,d$, $d\,e$, $e\,f$, and to the radius of the curve, as per our Table of Long Chords, p. 58.

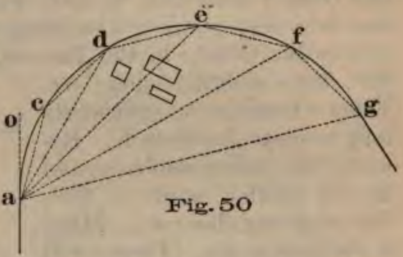

Fig. 50

Or, after sighting from a to f, measure from d to f a long chord $d\,f$ (not shown in the fig.) corresponding to *two* stations $d\,e$, $e\,f$, as per the same table, fixing the end f by the instrument at a. The curve may then be continued from f by tangential angles, $f\,a\,g$, etc., and 100 ft. chords, $f\,g$, etc., as at first.

Mode 4.—To carry either a tangent, $b\,f$, Fig. 51, or any curve (whether simple, compound, or reverse), past an obstacle o.

Running a line from k we meet the obstacle o. To pass it, select any point, a, in the line (which is here a curve), as near as convenient to the obstacle. Knowing the distance, $a\ o$, in chords, find the central angle, $x' =$ number of chords × chord-deflection angle.

Find the apex distance, $a\ b$ or $o\ b =$ radius $c\ a \times$ tangent of *half* x'.

Place the instrument at a; lay off a tangent $a\ b$; upon it measure off the apex distance $a\ b$; and drive a stake at b.

Then b is a point in the tangent $b\ f$.

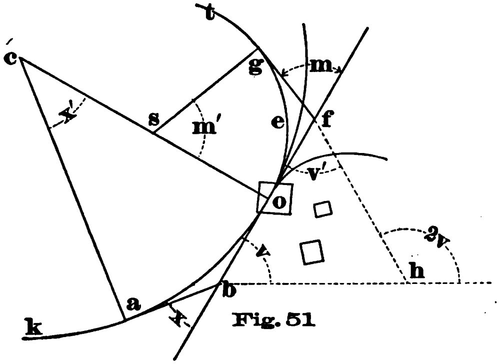

Fig. 51

Remove the instrument to b; take sight on a, and lay off the angle $x = x'$. Revolve the telescope, thereby sighting in the direction of the tangent $b\ f$.

Lay off, on either side of $b\ f$, any angle v (which had better, if possible, be just 60°), and any measured distance, $b\ h$, that will serve to clear the obstacle. Drive a stake at h.

Remove the instrument to h; take sight on b; revolve the telescope; lay off the angle $2v =$ twice v, thus sighting along $h\ f$.

Make $h\ f = b\ h$. Drive a stake at f, which will then be a point in the tangent $b\ f$.

If the angle v has been made 60°, $f\ b$ will be $= h\ f$ or $b\ h$; otherwise we must *calculate* $f\ b$, which is $= b\ h \times$ cosine of $v \times 2$.

Remove the instrument to f; take sight on h, and lay off the angle $v' = v$; *thereby* sighting along the tangent f b.

Now, if the curve ends at *o*, this tangent is, in itself, all we want; but if the curve (either simple, compound, or reverse, as represented by the three curves beyond *o*) is to be continued beyond *o*, we proceed in either of the three cases, and by the same process, first, to find a point in the curve beyond *o;* and then to run the curve in either or both directions from that point. For illustration we shall select the curve *o t*, and show how to find a point, *g*, in it.

On the tangent *f b* take any convenient point, as *f*, *at a known distance*, *f o*, from *o*.* Divide this *f o* by the radius, *o s*, of the curve to be run. The quotient will be a natural tangent. From the table take out the angle corresponding to it. Multiply this angle by 2. The product will be the central angle, m', of the part curve, *o e g*. Place the instrument at *f;* lay off m ($= m'$) thus sighting in the direction *f g;* and make *f g* equal to the known distance *f o*. Then is *g* a point in the curve; and *f g* is a tangent to the curve at *g*. Hence it is easy to run the curve in either direction from *g*.

Mode 5 accomplishes all that can be done by Mode 4, even in case some obstruction, as the river in Fig. 52, should prevent our making use of the apex point, *b*, so essential in that Mode. Thus if, when running from *k* towards *f*, we meet an obstacle, *o*, we *imagine* a stake, *f*, to be driven on the tangent, *at a given distance*, *f o*, from and beyond the obstacle, such that *f* could be *seen*, and *a f measured* from *a*. We then calculate the angle *b a f*, and the distance *a f*, both of which being laid off from *a*, plainly enables us to place an *actual* stake, *f*, in the tangent.

In the first place, to find the angle *b a f*, knowing the length in chords of the part curve, *a e o*, we can find its central angle, *x*, and its apex distance, *b a* or *b o;* also the angle *a b f* = 180° — *x*. Therefore, in the triangle *a b f*, we have given two sides, *a b* and *b f* (the last = *b o* + the given distance, *f o*)*;* and the angle *a b f;* to find the angle *b a f* and the side *a f*.

Now we know the sum of the two angles *b a f* and *b f a* to be = 180° — *a b f*. Call this sum *s;* and call their unknown difference, *d*. After finding *d*, then half of *s* +

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* If we select the point *f*, just found, we know that *f o* is = *f b* — *o b*.

half of *d* will equal the *larger* angle (*b a f*); and *half* of *s* — *half* of *d* will equal the *smaller* one (*b f a*), which also will be needed after finding *f*.

Now to find *d*, we use the well-known trigonometrical rule,

Sum of the two given sides : Their difference : : Tang. of half of *s* : Tang. of half of *d*.

Having in this way found the angle *b a f*, find the distance *a f*, by another familiar trigonometrical rule,

Sine of *b f a*, opposite the given side, *a b* : Sine of *a b f*, opposite the reqd. side, *a f* : : The given side, *a b* : The reqd. side, *a f*.

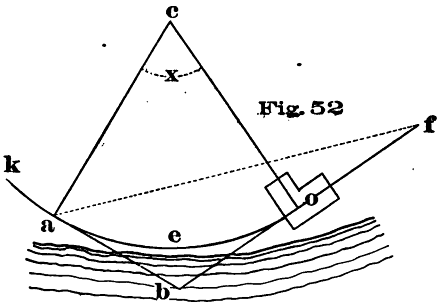
Fig. 52

Having found and measured off the distance, *a f*, and driven a stake at *f*, move the instrument to that point; take sight on *a*, and lay off the angle *b f a*. The telescope will then sight along the required tangent, *f b*.

From any point on this tangent any kind of curve may be commenced, if required, or may be continued from the point *o*, as in Fig. 51, Mode 4.

ARTICLE XXXVIII.

The object in easing off the ends of curves, that is, in gradually increasing the radius near the ends, is to enable trains to leave the straight line less abruptly, and thus to reduce the momentary *jar* so commonly felt at the instant of entering a curve, especially when of short radius, and at high speed. This jar, which is not only unpleasant to passengers, but damaging to engines and cars, is owing to the flanges at first *striking an oblique blow* against the outer rail of the curve. After this *blow* has been struck, the subsequent steady *pressure* against the outer rail is comparatively harmless; and indeed may be greatly increased by a gradual sharpening of the curve, without any repetition of jar.

The great point, therefore, is to carry the train from the straight line to the sharpest part of the curve, *with a steadily increasing pressure*, and without jar.

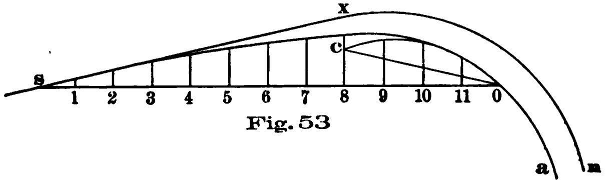

Fig. 53

The writer believes that the following simple process will be found to answer sufficiently well *in practice*, although many may consider it very crude *in theory*, inasmuch as it always gives the same distance for effecting the change.

Its execution in the field is extremely simple, which is an important point in such matters. It applies to the sharpest curve that can be traced with 100 ft. chords, as well as to the flattest one in which easing off of the ends will be at all likely to be considered expedient; say one of from 3,000 to 4,000 feet radius.

Rule.—Let $x\,n$, Fig. 53, be part of the curve; and $x\,s$ its tangent. Divide the chord-deflection distance (Table, p. 18) by 10. The quotient will be $c\,x$. Set every stake of the entire curve inward this distance, $c\,x$, so that the curve shall be removed to *c a*.* The radius of the curve is thus evidently shortened to the extent of *c x;* but this is of no importance. From *x* measure on the tangent 100 feet to *s;* and from *c* measure 50 feet to the curve at *o*.

Either stretching a piece of twine from *s* to *o*, or ranging along *s o* with the transit, lay off the 11 equidistant ordinates, if for rail laying; or only the middle one (6), or it and the two quarter-way ones (3 and 9), if for grading.

Since *x s* is always 100 feet, and *c o* 50 feet, the distance apart of these 11 ordinates will always be nearly 12·5 feet.

The ordinates themselves in feet are found for any curve, by multiplying *c x* by the following

MULTIPLIERS.

Ord.	Mult.	Ord.	Mult.	Ord.	Mult.	Ord.	Mult.
1	·180	4	·645	7	·975	10	·715
2	·355	5	·775	8	·990	11	·430
3	·505	6	·890	9	·905		

Example.—Let *x n*, Fig. 53, be part of a 6° curve, or 955·4 radius, and *x s* its tangent. Its deflection distance (Table, p. 20) is then 10·47 feet; one-tenth of which, or 1·047 feet, is *c x*. Move all the stakes of the entire curve inward 1·047 feet, as along *c o a*.† From *x* measure 100 feet to *s;* and from *c* measure *c o*, 50 feet, to the curve; driving stakes at *s* and *o*. Multiply *c x*, or 1·047 feet, by the above multipliers; thus finding the following 11 ordinates, of which lay off as many as are needed.

---

* **Or, which would be much better,** the curves may be traced inside of their tangents during the definite location, thus avoiding the necessity for removing them, or for shortening the radius.

† **The table of** *c x* **in feet below** shows that even very sharp curves require to be moved inward but a short distance.

Deg.	Rad. Feet.	c x. Feet.	Deg.	Rad. Feet.	c x. Feet.	Deg.	Rad. Feet.	c x. Feet.
19°	303	3·30	8¼°	695	1·44	4°	1433	·70
14	410	2·44	7	819	1·22	3	1910	·52
11½	499	2·00	5¾	997	1·00	2	2865	·35
9½	604	1·66	4¾	1207	·83	1	5730	·17

Ord.	Ord.	Ord.	Ord.
1.=·188	4.=·675	7.=1·021	10.=·749
2.=·372	5.=·811	8.=1·037	11.=·450
3.=·529	6.=·932	9.= ·948	

ARTICLE XXXIX.

Resistance of Curves.

The following will merely give some general idea of the principles involved in investigating this most intricate and *practically unsolvable* subject.*

The resistance which curves oppose to the passage of trains is influenced by many circumstances: such as the velocity; radius of the curve; wind; diameter of the wheels; shape of the wheel-treads, whether more or less conical; by the distances apart of the several pairs of wheels; by whether the cars are empty or loaded, for an empty train offers greater resistance than a loaded one *of the same weight;* by the kind of coupling; by the width of track; its condition; the degree of elevation of the outer rail; the length of the train, for a long train experiences proportionately more resistance than a short one, owing to the obliquity of the traction, etc., etc. From the want of sufficient experimental data, the subject is but imperfectly understood, and consequently all calculated results are more or less liable to error. Our own opinion is that the resistance has been usually underrated. So far as we can judge from the incomplete and contradictory experiments and observations that have been recorded, we are inclined to believe that at the speed of about 25 miles per hour, on a track in good order, and with trains equal to from 6 to 10 eight-wheeled cars, we may assume, as a rough approximation, that a level curve of 400 feet radius will oppose a resistance of about 15 lbs. for every ton weight of the train, *in addition* to the resistance on a level straight line. On a level straight line, *in good order*, and in calm weather, the resistance at 25 miles per hour is about 12 lbs. per ton. On these assumptions, therefore, the *total* resistance on a level curve of 400 feet radius, at 25 miles per hour, would be 27 lbs. per ton, or 2¼ times as great as on a straight line.

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* Of late years much has been written on the subject of the influence of curves and grades; by none more ably than by Mr. Arthur M. Wellington, Civ. Eng., to whose "Economic Theory of the Location of Railways," published by the "Railroad Gazette," 73 Broadway, N. Y., we refer those desiring information.

Now, the assumed additional 15 lbs. per ton, due to curvature, or deflection, alone, without any regard to distance, is the $\frac{1}{149\frac{1}{3}}$ part of a ton; and the $\frac{1}{149\frac{1}{3}}$ part of a mile is 35·36 feet; being that grade, or inclination, which, by the principle of the inclined plane, increases the *gravity resistance* of a ton, or of any other weight, the $\frac{1}{149\frac{1}{3}}$ part. Therefore, our ascribed resistance for a radius of 400 feet, at 25 miles per hour, is equal to that of an upgrade of 35·36 feet per mile; or, ·670 of a foot per 100 feet. If, in addition, we assume, as is usual, but probably incorrect, that at the same speed the resistance of curves is as the tabular angle of deflection, or *inversely* as the radius (although some experiments seem to show that the increase is much more rapid), we arrive at the results given in the following table; and which are probably at best but rude approximations. They are about one-sixth part greater than our suggestion for reduction in the Table on p. 95. Which is nearest the truth, we are unable to say.

TABLE OF RESISTANCES

(IN EXCESS OF THOSE ON A STRAIGHT LINE) DUE TO CURVATURE ALONE; AT A VELOCITY OF 25 MILES PER HOUR.

For the *total* approximate resistance, add 12 lbs. to each amount in the third column; 28·4 feet to each grade per mile; and ·538 of a foot to each grade per 100 feet.

Radius.	Approx. ang. of Def.		Resist. per ton, from curvature alone.	Equivalent upgrade; in feet.		Radius.	Approx. ang. of Def.		Resist. per ton, from curvature alone.	Equivalent upgrade; in feet.	
Feet.	D.	M.	Pds.	Per mile.	Per 100 ft.	Feet.	D.	M.	Pds.	Per mile.	Per 100 ft.
400	14	22	15	35·36	·670	2400	2	23	2·5	5·89	·112
500	11	30	12	28·29	·536	2600	2	12	2·31	5·44	·103
600	9	34	10	23·57	·447	2800	2	2	2·14	5·05	·096
700	8	12	8·57	20·21	·383	3000	1	54	2	4·71	·089
800	7	13	7·5	17·68	·335	3200	1	47	1·88	4·42	·084
900	6	22	6·66	15·72	·298	3400	1	41	1·76	4·16	·079
1000	5	43	6	14·14	·268	3600	1	35	1·67	3·93	·075
1100	5	12	5·45	12·86	·244	3800	1	30	1·58	3·72	·071
1200	4	47	5	11·78	·224	4000	1	26	1·50	3·54	·067
1300	4	25	4·64	10·94	·208	4200	1	22	1·43	3·38	·064
1400	4	5	4·28	10·11	·192	4400	1	18	1·36	3·22	·061
1500	3	49	4·02	9·48	·180	4600	1	15	1·30	3·08	·058
1600	3	35	3·75	8·84	·168	4800	1	12	1·25	2·94	·056
1800	3	11	3·33	7·86	·149	5280	1	5	1·14	2·68	·051
2000	2	52	3	7·07	·134	5730	1	0	1·05	2·47	·047
2200	2	36	2·72	6·43	·122	6000	0	57	1	2·36	·044

For ease of recollection, we may, according to this, consider the resistance of curvature alone, at 25 miles per hour, as about 1 lb. per ton for each degree of chord deflection angle. If, on ascending grades, we wish to equalize the traction on curves, and on straight lines, for a speed of 25 miles per hour, we must flatten the grades on the curves at the rates given in the last two columns. But since the resistance of curvature is affected by the velocity, it plainly follows that the flattening which is best for fast passenger trains, cannot also be best for slow freight; so that, even if we could determine a precise law of resistance, we could not so apply it as to suit both. So far as we are aware, there are no absolutely reliable data on the additional resistance of curves at different speeds.

Some have supposed it to be independent of the velocity; but it has generally been assumed to *increase* in some proportion to it; but Mr. Wellington found "that the additional resistance of a 1° curve is over 1 lb. per ton (2000 lbs.) at a velocity of 12 miles per hour, and *decreases* to about ½ lb. per ton at a velocity of 22 miles per hour. The resistance of an 8° curve was over 8 lbs. per ton at a velocity of 9 miles per hour, and *decreased* to about 6½ lbs. per ton (probably) at a speed of 19 miles per hour."

These unexpected results are in accord with Professor Thurston's discovery that journal friction decreases at high velocities.

On the Pennsylvania R. R. the grades were originally flattened on curves, at the rate of from full 1 foot per mile, on moderate ones, to 1¼ foot on the steeper ones, for each 1° of chord deflection angle of the curve; but experience on that road at speeds less than 25 miles per hour showed this to be too little.

Perhaps until we can *with certainty* do better, we may, for any velocity, reduce the grade on curves at the rate of ·04 of a foot per 100 feet (2·112 ft. per mile), for each degree of chord angle of deflection (p. 18), as shown in the last two columns of the following table; or about one-seventh part less than in the preceding one.

Remark.—Since the reduction of grade is often difficult to accomplish without much expense, it seems scarcely worth while to do it at all in such cases (or indeed in others), except when, without it, the curves would tax the power of the locomotives unduly.

TABLE FOR THE REDUCTION OF GRADES ON CURVES.

Deg.	Rad.	Per Mile.	Per 100 ft.	Deg.	Rad.	Per Mile.	Per 100 ft.	Deg.	Rad.	Per Mile.	Per 100 ft.
1	5730	2·112	·04	9	637	19·008	·36	17	338	35·904	·68
2	2865	4·224	·08	10	574	21·120	·40	18	320	38·018	·72
3	1910	6·336	·12	11	522	23·232	·44	19	303	40·128	·76
4	1433	8·448	·16	12	478	25·344	·48	20	288	42·240	·80
5	1146	10·560	·20	13	442	27·456	·52	21	274	44·352	·84
6	955	12·672	·24	14	410	29·568	·56	22	262	46·464	·88
7	819	14·784	·28	15	383	31·680	·60	23	251	48·576	·92
8	717	16·896	·32	16	359	33·792	·64	24	241	50·688	·96

Remark.—It is plain that, when the grades are very moderate, and the curves at the same time sharp, **the grades cannot be reduced sufficiently to compensate for curvature**; so also when the road is level.

Many of the principal railroads, both in the United States and in Europe, have some curves with very small radii. On the Pennsylvania R. R. are several, of from 637 to 717 feet radius. The Reading has several of 800 feet. On the New Jersey Central is one of 400 feet. On the Baltimore and Ohio are many of from 400 to 600 feet. On all these roads, however, the curvature is being reduced at heavy expense. On the Lehigh Valley road is a curve of nearly a semicircle, the ends of which have radii of 717 feet, thence gradually decreasing both ways toward the center, where it is only 359 feet. About half this curve has a grade of 10 feet per mile, and the other half has 30 feet; both in the same direction. Descending trains of 110 four-wheeled loaded coal-cars (900 tons in all) have no difficulty in passing the curve; but if the empty trains stop on ascending, they frequently have great trouble to start again, and then resort to sanding the rails.

On the Mahanoy and Broad Mountain road, tank engines of 35 tons, all on 8-drivers, draw 40 empty coal-cars, weighing 100 tons, up a continuous grade of 175 feet to a mile, for 3½ miles, around an almost constant succession of curves of from 450 to 600 feet radius, at 8 miles an hour, as a regular business.

When the radius is less than about 1000 feet, **the width of the track should be slightly increased,** otherwise the wheels of the train are apt to bind between the rails and break. About an inch will answer for this purpose on a curve of 400 feet radius; ¾ inch for 600; ½ inch for 800; ¼ inch for 1000, as a purely empirical approximation.

Curves are especially objectionable in deep cuts and on steep grades. In the former they prevent the driver from seeing ahead; and when descending the latter there is greater danger of leaving the track, inasmuch as the speed of the engine is not under as perfect control; especially with slippery rails.

The coning of the treads of the wheels tends theoretically to diminish the resistance of curves, by virtually enlarging the diameter of the outer wheels to a degree commensurate with the greater distance they have to travel along the outer rail. **The elevation of the outer rail,** by counteracting the centrifugal force, still further reduces the resistance. But unfortunately these aids cannot be so applied as to suit the different velocities of fast and slow trains. If, as is always done (with a view to the safety of passengers), they are adapted for fast trains, then they produce an opposite effect upon the slow ones, which, for want of sufficient centrifugal force, slide *down* the inclined plane formed between the two rails, until the lower flanges rub against the inner rail. When this takes place, the inner wheels not only have a less distance to travel than the outer ones, but, their diameter becoming enlarged, they must evidently *slide*, as well as revolve, in order to keep pace with them. This sliding produces a dangerous twisting, or torsional, strain upon the axles, rendering them liable to break, especially if the cars are heavily loaded.

Moreover, even when the coning of the treads enables the cars to run more easily around *curves*, it adds to the resistance upon *straight lines*. Inequalities of the track then cause the train to run in a zigzag line, pressing the flanges alternately against the opposite lines of rails. On this account the coning has of late years been much reduced, until at present it rarely exceeds 1 in 20; and on some of our principal roads it is but 1 in 50. Finally, the cone is soon worn off by use, and the wheels become cylindrical.

ARTICLE XL.

On the Elevation of the Outer Rail on Curves.

When a train is going around a curve, the centrifugal force throws it *outward* against the outer rail; and this force increases *directly* as the square of the speed, and *inversely* as the radius. To counteract it, an *inward* tendency is given to the train by placing it, as it were, upon an inclined plane formed by raising the outer rail above the inner one.

It is evident that theoretically each velocity requires its corresponding elevation; but inasmuch as this cannot be effected in *practice*, the elevation is proportioned to the *greatest probable* speed, in order to secure the safety of passengers. Slow freight trains will then slide *down* the inclined plane; and the flanges of their wheels will rub against the inner rail, and wear it more rapidly than the outer one; but this must be submitted to.

On the other hand, a great elevation of the outer rail causes the cars to lean sideways to a degree that is disagreeable to passengers; and liable to displace freight. Therefore a limit of about 6 inches on 4 ft. 8½ in. gauge is usually assumed as the greatest elevation to be given *in any case;* and where the curves are so sharp that this is not enough for safety at great speed, orders are given to diminish the speed. This would answer very well if the orders could always be enforced; but as this cannot be done, it involves an element of danger that can be averted only by the adoption of easy curves.

Either of the two following formulas gives the elevations in the next table. Both might be greatly and uselessly complicated by admitting the coning of the wheel and other considerations; but these refinements may safely be discarded.

Formula 1.—$$\text{Elevation in inches} = \frac{\text{Square of speed in feet per second} \times \text{gauge in inches}}{\text{Radius in feet} \times 32{\cdot}2}.$$

Formula 2.—$$\text{Elevation in inches} = \frac{\text{Square of speed in miles per hour} \times \text{gauge in feet}}{\text{Radius in feet} \times 1{\cdot}25}.$$

The common gauge of 4 ft. 8½ ins. is equal to 4·7083 feet, or to 56·5 inches.

Remarks.—While speed was restricted to about 35 or 40 miles per hour, the rule of thumb, of half an inch elevation for each degree of chord deflection angle, seems to have answered very well, although but **half of what the formulas require,** as seen in our table, in the column for 40 miles speed. But of late years this has, on many leading roads, been increased to 1 inch per degree of chord deflection angle, to meet the increased speeds of 50, 60, or more miles per hour, which are becoming not unfrequent; more than 80 have in fact been accomplished. The maximum elevation, however, is still limited to about 6 inches on 4 ft. 8½ in. gauge.

It will be seen that on **any** curve in our table (from 1° to 40°, or from 5730 to 146 ft. radius), the rule of 1 inch per degree is safe at a speed of nearly 40 miles per hour; but that after 6°, or with less radius than 955 feet, the above limit of 6 inches is exceeded. There can, however, be no doubt that where the elevation has been but ½ an inch per degree, trains have daily travelled curves at 40, and, perhaps, at times, at 50 or more miles per hour; and that **even where there was no elevation at all,** but where the formulas call for about 3 to 3½ inches, as on turnouts of only 700 to 800 ft. radius, they have habitually run at 25 or more miles per hour.

These facts, however, do not invalidate the principle of the formulas.

Our following table indicates that at 1 inch per degree, with a limit of 6½ inches total elevation, a 1° curve would be safe at 100 miles an hour; a 2° one at 70 miles; a 3° one at near 60; a 4° one (1433 ft. rad.) at 50; a 6° one (955 ft. rad.) at 40; and a 10° one (574 ft. rad.) at full 30 miles.

Our table is for the standard gauge of 4 ft. 8½ ins.; for greater or for less gauges the elevation will increase or diminish directly as the gauge; thus maintaining the same rate, or angle of elevation in all cases.

The elevation must, of course, be made gradually.

When the curve is uniform, that is, when its ends are not eased off by larger radii, it is usual to begin the rise of the outer rail at a distance of from 50 to 100 feet back on the straight line, **for each inch of elevation.** Thus, for 6 inches elevation, some engineers go back 600 feet, and others but 300, and rise gradually until the entire elevation is attained by the time they reach the P C, or beginning of the curve.

But if easement curves are used at the ends of the main one, the elevation is begun at the beginning of the easement.

The writer believes that even 50 feet per inch of elevation is more than is *necessary*. In his suggestion, p. 90, for using easement curves 150 feet long in all cases, if the elevation begins with the easement, it will (for a speed of 60 miles per hour) vary between 2·37 inches in 150 feet on a 1° curve (5730 ft. rad.); and 6 inches in 150 feet on all curves of 2½° or more, supposing 6 inches to be the limit. This last is equal to 1 inch in 25 feet, or to a grade of 17·6 feet per mile, which the writer cannot regard as excessive, especially when the grades are reduced for curvature.

In the Wharton Safety Switch, as it has been laid for many years on a number of our main lines, there is an elevation of 2½ ins. in a distance of only 4 feet; or at the rate of 15·6 ins. in 25 feet; or 275 feet per mile. This sudden rise has since been reduced to 1¾ ins., or still nearly 11 times our maximum rate.

Should ours, however, be considered too rapid a rise, the elevation may be commenced 50 or more feet farther back on the tangents of such curves as require more than say about 4 inches elevation; and without any change in the easement curvature itself. But, as before remarked, the writer does not himself consider this at all necessary; the main point being, in his opinion, to **enter** the easement curve **without jar**; and then to maintain a gradually increasing outward **pressure** (insensible to passengers) until the main curve is reached. He believes that by the foregoing method these desiderata will be secured, at least so far as practical considerations may call for.

TABLE OF ELEVATION OF OUTER RAIL IN CURVES; FOR GAUGE 4 FT. 8½ INS.

Angle of Deflection.	Radius in Feet.	VELOCITY IN MILES PER HOUR.										
		5	10	15	20	30	40	50	60	70	80	100
		VELOCITY IN FEET PER SECOND.										
		7·33	14·7	22·0	29·3	44·0	58·7	73·3	88·0	103	117	147
		SQUARE OF VELOCITY IN FEET PER SECOND.										
		53·8	215	484	860	1936	3442	5378	7744	10540	13766	21510
		ELEVATION OF OUTER RAIL IN INCHES.										
1°	5730	·02	·07	·15	·26	·59	1·05	1·65	2·37	3·23	4·22	6·59
2°	2865	·03	·13	·30	·53	1·19	2·11	3·29	4·74	6·45	8·43	13·2
3°	1910	·05	·20	·45	·79	1·78	3·16	4·94	7·11	9·68	12·6	19·8
4°	1433	·07	·26	·59	1·05	2·37	4·22	6·59	9·48	12·9	16·9	26·3
5°	1146	·08	·33	·74	1·32	2·46	5·27	8·23	11·9	16·1	21·1	
6°	955	·10	·40	·89	1·58	3·56	6·32	9·88	14·2	19·4		
7°	819	·12	·46	1·04	1·84	4·15	7·38	11·5	16·6	22·6		
8°	717	·13	·53	1·19	2·11	4·74	8·43	13·8	19·0			
9°	637	·15	·59	1·33	2·37	5·34	9·49	14·8	21·3			
10°	574	·17	·66	1·48	2·63	5·93	10·5	16·5	23·7			
12°	478	·20	·79	1·78	3·16	7·11	12·6	19·8				
14°	410	·23	.92	2·07	3·68	8·29	14·7	23·0				
16°	359	·26	1·05	2·37	4·21	9·46	16·8					
18°	320	·30	1·18	2·65	4·72	10·6	18·9					
20°	288	·33	1·31	2·95	5·24	11·8	21·0					
25°	231	·41	1·63	3·68	6·54	14·7	26·3					
30°	193	·49	1·96	4·40	7·82	17·6	31·6					
35°	166	·57	2·27	5·12	9·10	20·5	36·9					
40°	146	·65	2·59	5·82	10·3	23·3	42·2					

ARTICLE XLI.

Equation of Curvature.

This consists in ascertaining what amount of *straight distance* produces an expenditure of motive power equal to that produced by a given amount of curvature, say 1°.

There is a certain amount of resistance encountered by trains on a straight line; and the overcoming of this resistance costs money, not only for motive power, but for repairs of engines, cars, tracks, bridges, etc. But if that identical piece of straight road be bent into a curve, without any alteration in its length, then the resistance and the consequent expense of motive power and repairs will also be increased; and it is usually supposed that the increase will be in proportion to the amount of bending. This increase, therefore, is plainly not a consequence of the *distance*, which remains as before, but merely of the *bending, deflection*, or *curvature;* and in equating for curvature, with a view to a comparison with straight lines, we have to consider, not the total resistance upon the *curve*, but only that portion of it which is due to the *curvature*. If we knew, from experiment and observation, how much the expenses of running a road are affected by curvature, we might prepare formulas giving a tolerably accurate solution of the question; but in the absence of such data, we are compelled to resort to certain assumptions, the correctness of which is somewhat problematical. It is probable, also, that facts which should materially modify our conclusions are lost sight of; as, for example, the greater *danger* of sudden curves.

It is assumed that the *total amount* of *extra* power due to *curvature*, and expended in running around any given curve, at any given speed, is in proportion to the number of *degrees* contained in the curve, without regard to its *radius* or *length*.

Thus, 1° of radius of 400 feet has only ¼ the length of 1° of 1600 feet radius; but the *extra* power exerted *at any one instant*, on the short radius, must be 4 times as great as that on the long radius, in order to keep up the same speed on both. But on the long radius, although the power exerted at any one instant is only ¼ as great as that on the short one, still it has to be exerted 4 times as long, or during 4 times as many instants, while carrying the train at the same rate of speed through its 4-times-as-long 1°. Therefore, the total expenditure of *extra* power in running around 1° of curve is the same in both cases.

Now, we have already said that on a level straight line, *in perfect order*, and with the machinery in ordinary use, the resistance to a train moving at 25 miles an hour may be taken as approximately equal to 12 lbs. per ton of the entire weight of the train. We have also assumed that a curve with 1° of deflection angle *increases* this resistance to the extent of 1·05 lbs.; and that with other angles of deflection, the *increase* of resistance will be in proportion to the number of degrees contained in them; so that a curve of 11½° deflection angle will present an *increase* of continuous resistance equal to the whole of that on the straight line. In other words, the *total* resistance exerted at each instant on the curve will be twice that exerted on the straight line at the same speed. Hence, if on any proposed lines of survey we have a mile of 11½° curvature, upon which the velocity is to be 25 miles per hour, that mile will require as much power as 2 miles of straight line. But a curve of 11½° deflection angle has a radius of 500 feet, and a circumference of 3141 feet, which latter is of course equal to 360° of the curve. And if 3141 feet are equal to 360°; a mile, or 5280 feet, is equal to 605°. Hence, 605° of curve of 11½° deflection angle requires a *total* expenditure of power equal to that required on 2 miles of straight line; in other words, the curvature alone requires an *increase* of power equal to the *total* power required on a mile of straight line. Therefore, if this mile, or 605° of 11½° curve, could be straightened into a mile of direct line, we should forever afterwards save the expense of half the power required to run it; that is, we should save power enough to run one mile of straight line. But we have before assumed that the power expended upon curves is in proportion to the number of *degrees* contained in the entire curve, without any regard to the radius, or to the length of the curve. If this be the case, it follows that, by merely *straightening* 605° of *any* curve, we shall, without diminishing its length, save power enough to run 1 mile, or 5280 feet of straight line; or by straightening 1° we shall save power sufficient for $\frac{5280 \text{ feet}}{605°} = 8{\cdot}7$ feet of straight line; and with this power, we should also save the wear and tear of machinery and track, etc., which it produces, and which are assumed to be about in proportion to the power expended.

But the important point is to reduce this saving of power and repairs to a *moneyed* value. This will vary with the annual expense of running the road. The process usually adopted for this purpose is as follows: Experience shows that of all the annual expenses of running a railroad, those which may be assumed to be pretty nearly in proportion to the power expended, such as wear and repairs of engines, cars, track, etc., etc., compose, as an average of many roads, about two-thirds. Therefore, if we judge from previous calculations that the annual receipts on our proposed railroad will be about $4500 per mile; and the expenses about $3000, or two-thirds of the receipts, which is the approximate average of most railroads; then about $2000 per mile, or 38 cents per foot of road, will generally be nearly in proportion to the motive power expended. But we have seen that 1° of curvature, or deflection, incurs as much annual expense for motive power as 8·7 feet of straight line; or, in this case, 38 cts. × 8·7 = $3·30. Now, 38 cents is the interest at 6 per cent. on $6·33; and $3·30 is the interest on $55; therefore, in this instance, according to the foregoing, and with a speed of 25 miles per hour, we should be warranted in expending $6·33 to shorten the length of the road one foot; or $55 to merely *straighten* 1° of curvature.

Having arrived thus far, we are enabled to decide, to some extent, upon the comparative merits of two or more surveyed routes for our road; that is, we can *equate them for curvature.* Thus, suppose that one of the surveys is 100 miles long, and has 3025° of curvature; while the other is but 98 miles long, but has 4840° of curvature. Now, since the annual expenses of 605° of curvature are equal to those on a mile of straight line, we have for the two lines as follows:

	Miles of distance.		Miles equated for curvature.		Miles as regards annual expenses of running the road.
1st line.	100	$+ \left(\frac{3025^\circ}{605^\circ} = \right)$	5	=	105
2d line.	98	$+ \left(\frac{4840^\circ}{605^\circ} = \right)$	8	=	106

Therefore, as regards annual expenses, the longest line will be the cheapest by nearly 1 per cent., so far as curvature is concerned. This may, however, be neutralized by superiority of grades on the shorter line; or by other causes not of an engineering character.

This, we believe, is about the view usually taken of this subject. Engineers, however, generally assume the resistance of curves at much less than our estimation of it, and consequently give a shorter straight distance as equivalent to 1° of curvature. Although we regard the whole process as empirical, it at least serves to caution us against too hastily introducing curves, from a mistaken idea of economy in the first outlay. On the Pennsylvania R. R., at the time of its location, the saving of 1 foot of distance was valued at \$10, or \$52800 per mile; and the saving of 1° of curvature at \$50, or \$18000 for a complete circle; thus making 1056°, or nearly three complete circles to be equivalent to 1 mile of distance. With the present enormous business of that road the foregoing valuations of curvature and distance would be absurdly small. Competition is a powerful element in such matters.

Finally, inasmuch as the foregoing is merely a crude, incomplete, and superficial treatment of this difficult subject, we again refer those who wish to study it in the light of the most recent experience and investigation, to the standard "Economic Theory of the Location of Railways," by Arthur M. Wellington, C. E.

TABLE OF MIDDLE ORDINATES.

To be used for the bending of rails of different lengths, so as to form portions of curves of different radii. Ordinates for lengths or radii intermediate of those in the table, may be found by simple proportion.

Def. ang.	Radius.	LENGTHS OF RAILS.												
		30	28	26	24	22	20	18	16	14	12	10	8	6
Deg.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
·5	11460	·010	·008	·006	·005	·004	·004	·003	·002	·002	·001	·001	·000	·000
1·	5730	·020	·016	·013	·011	·009	·008	·006	·005	·004	·003	·002	·001	·001
1·5	3820	·029	·026	·021	·018	·016	·013	·010	·008	·006	·004	·003	·002	·001
2·	2865	·038	·034	·029	·025	·021	·017	·014	·011	·008	·006	·004	·003	·001
2·5	2292	·049	·043	·037	·031	·027	·022	·018	·014	.010	·007	·005	·003	·002
3·	1910	·058	·051	·044	·037	·031	·026	·022	·017	·012	·009	·006	·004	·002
3·5	1637	·070	·061	·052	·043	·037	·031	·025	·020	·015	·011	·008	·005	·003
4·	1433	·079	·069	·060	·050	·042	·035	·029	·023	·018	·013	·009	·006	·003
4·5	1274	·088	·077	·067	·056	·047	·039	·032	·026	·020	·015	·010	·007	·004
5·	1146	·099	·086	·074	·063	·053	·044	·035	·029	·022	·016	·011	·007	·004
5·5	1042	·108	·094	·082	·070	·059	·048	·039	·032	·024	·018	·012	·008	·004
6·	955·4	·117	·102	·088	·076	·064	·052	·042	·034	·026	·019	·013	·008	·005
6·5	882	·128	·112	·097	·082	·069	·057	·046	·037	·028	·021	·014	·009	·005
7·	819	·137	·120	·104	·088	·074	·061	·049	·039	·030	·022	·015	·010	·005
7·5	764·5	·146	·127	·111	·094	·079	·065	·053	.042	·032	·024	·016	·010	·006
8·	716·8	·158	·137	·119	·100	·085	·070	·056	·045	·034	·025	·017	·011	·006
8·5	674·6	·166	·145	·126	·106	·090	·074	·060	·048	·036	·027	·018	·012	·007
9·	637·3	·175	·153	·133	·112	·095	·078	·063	·050	·038	·029	·019	·012	·007
9·5	603·8	·187	·163	·141	·119	·101	·083	·067	·054	·042	·031	·021	·013	·008

TABLE OF MIDDLE ORDINATES—CONTINUED.

Def. ang.	Radius.	LENGTHS OF RAILS.												
		30	28	26	24	22	20	18	16	14	12	10	8	6
Deg.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
10	573·7	·196	·171	·148	·125	·106	·087	·071	·057	·045	·032	·022	·014	·008
11	521·7	·216	·188	·163	·139	·117	·096	·078	·063	·049	·036	·024	·016	·009
12	478·3	·236	·206	·179	·151	·128	·105	·085	·069	·053	·039	·026	·017	·010
13	441·7	·254	·222	·192	·163	·138	·113	·092	·075	·057	·042	·028	·019	·010
14	410·3	·275	·239	·207	·175	·148	·122	·099	·080	·061	·045	·030	·020	·011
15	383·1	·295	·257	·223	·188	·159	·131	·106	·085	·065	·049	·033	·021	·012
16	359·3	·313	·273	·236	·200	·170	·139	·113	·091	·070	·052	·035	·023	·013
17	338·3	·333	·290	·252	·213	·180	·148	·120	·096	·074	·055	·037	·024	·014
18	319·6	·351	·306	·265	·225	·190	·156	·127	·102	·078	·058	·039	·025	·014
19	302·9	·371	·324	·280	·238	·201	·165	·134	·108	·082	·061	·041	·027	·015
20	287·9	·392	·341	·296	·250	·212	·174	·141	·114	·087	·066	·044	·028	·016
21	274·4	·410	·357	·309	·262	·222	·182	·148	·120	·091	·069	·046	·030	·017
22	262·	·430	·375	·325	·275	·233	·191	·155	·126	·096	·072	·048	·031	·018
23	250·8	·450	·390	·338	·287	·243	·199	·162	·131	·100	·075	·050	·033	·019
24	240·5	·469	·408	·354	·299	·253	·208	·169	·137	·104	·078	·052	·034	·019
25	231	·486	·424	·367	·311	·263	·216	·176	·142	·108	·081	·054	·035	·020
26	222·3	·506	·441	·382	·323	·274	·225	·183	·148	·112	·084	·056	·037	·021
27	214·2	·524	·457	·396	·335	·284	·283	·190	·153	·116	·087	·058	·038	·022
28	206·7	·545	·475	·411	·348	·294	·242	·197	·158	·120	·090	·060	·039	·022
29	199·7	·564	·491	·424	·361	·303	·250	·203	·163	·124	·093	·062	·041	·023

ARTICLE XLIII.

The Engineer's Transit.

The following description is longer than desirable; but it would have been much more so if we had not assumed that the reader has an actual transit before him, and can thus see at a glance many points which it would be tedious to describe in writing, and which we therefore omit.

The details of the Transit are differently arranged by different makers, and to suit special purposes. Still its essential parts so nearly resemble each other in all, that they may be understood from our Figs. 53¼ and 53½, which represent it in its modern form, as made by Heller & Brightly, of Philadelphia.* This widely known firm frequently modify the details of their instruments to meet the requirements of purchasers; so that in some cases they do not correspond exactly to the following description, or to our figs. We will specify some of the variations as we proceed. The letters on the two figs. correspond. Some letters are repeated for different parts, but not where they could lead to error.

The long bubble-tube, F F, Fig. 53½, under the telescope; and the **vertical graduated arc** *g*, are furnished only when the instrument is to be used for levelling or for measuring vertical angles. Without these appendages the instrument is their **Plain Transit.** With them, or rather with a graduated *full circle* instead of a mere *arc*, it becomes virtually a **Complete Theodolite,** vastly preferable to the clumsy and heavy instruments occasionally imported from Europe under that name.

Beginning at the **wooden legs,** their heads, Q, Fig. 53½, are attached (by means of bolts with wing heads) to lugs, D, cast in one with a stout circular piece, B, Fig. 53¼, called the **Tripod Head,** which screws up into the **lower parallel plate,** S. The screw-threads at *v* receive the screw of a wooden tripod head cover when the instrument is out of use.

Referring now to Fig. 53¼, in the center of the lower

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*The price of a first-class plain transit, with shifting-plate and plumb-bob, by these makers, is $185. One with vertical arc, *g*, and *long bubble-tube*, F F, $220.

parallel plate, S S, is seen a large circular opening, in which plays a casting, *c c d d*, the upper part, *d d*, of which, forms a socket enclosing the half ball, *b;* while its lower part, *c c*, below S, constitutes **the shifting plate.** The object of this shifting plate, and of the large opening, is, after the transit is set *very nearly* over the center of a stake, to allow it to be placed *exactly* over it, by shifting all the upper parts of the instrument a trifle, without moving the legs; thus saving time. To permit this shifting, the **levelling screws,** K, must first be a little loosened, but after the shifting they are tightened again, by which process they push upwards the **upper parallel plate,** *m*, thereby drawing upward the half ball, *b*, which in turn draws up the shifting plate, *c*, firmly against the lower side of S, and keeps it there.

Above the half ball, *b*, and screwed to the top of it, is the single casting, *m m m x x*, the upper part, *m m m*, of which, forms the **upper parallel plate;** while the part *x x* forms a socket into which the spindles, U, T, and *w*, foot. **The half ball,** by its play in its socket, *d d*, allows the upper parallel plate, *m m* (and all the upper parts of the instrument), to be set level by the levelling screws, K, although the *lower* parallel plate, S S (as constantly happens), may not be so. The **plumb-bob** string passes through the vertical hole seen in the center of the half ball.

The four levelling screws, K, are protected from rain and dust by screw caps, *f* and *g*, which may be removed as shown at the right-hand screw, K.

The feet, *i*, of the screws, work in loose sockets, *j*, which are flat at bottom, to preserve the plate, S, from being indented.

The parts thus far described are generally left attached to the wooden legs, not only in the field, but in the house between work. **The parts above** *m* (including the spindles, U, T, and *w*, and all the upper parts, which they support), may at any time be lifted together off from, or replaced upon, the parts below, thus:

To place the upper parts upon the parallel plates, place the lower end of the spindle, U U, in the socket, *x x*, holding the instrument so that the three recesses in the flange, *a a*, shall pass down *over the three* corresponding blocks, Y,

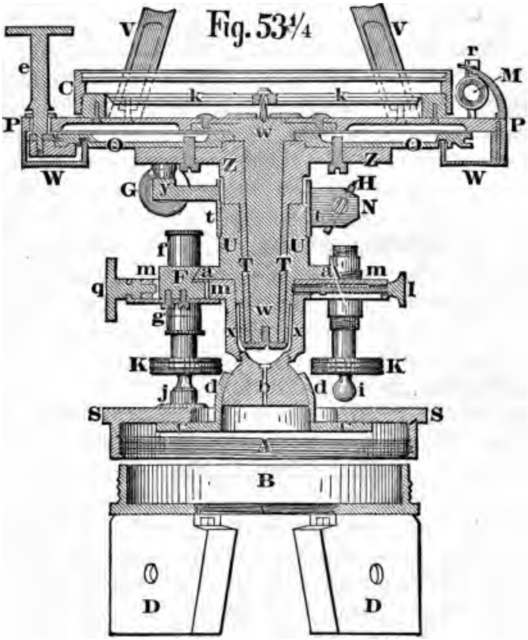
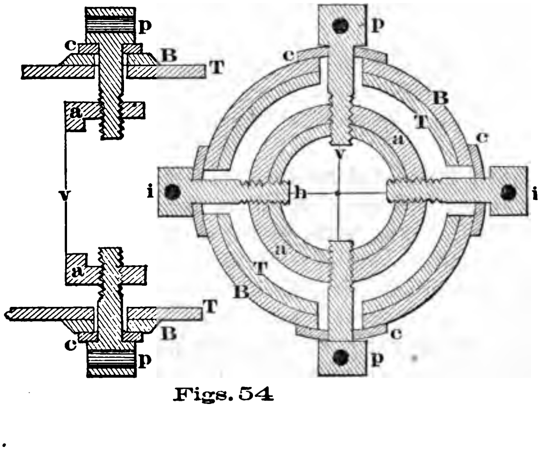


Fig. 53¼



Figs. 54

THE ENGINEER'S TRANSIT.

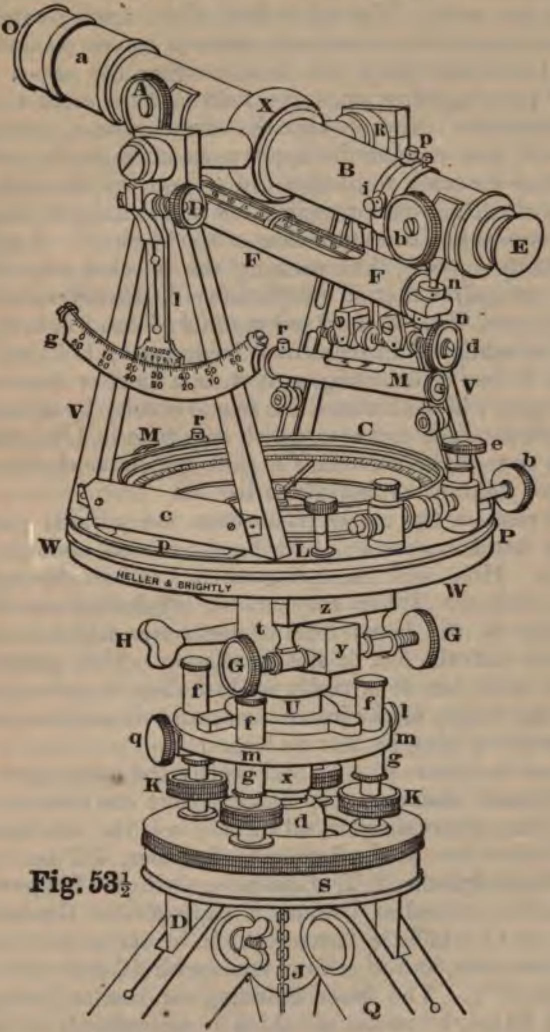


Fig. 53½

on the upper side of *m;* thus allowing the flange to bear fully upon *m*, and thus to bring upon *m* the weight of all the upper parts. The inner end of the **spring-catch,** *l*, in the meantime, automatically enters a groove around U, just below the flange *a a*, thus securing the upper and lower parts together when the instrument is carried over the shoulder. Now see that the clamp screws, *e* and H, are fast; and revolve the upper parts horizontally a trifle in either direction until they are stopped by the striking of the small lug on *a a*, against one of the fixed blocks, F. The recesses in *a a* are now clear of the blocks, F. Tighten the clamp screw *q*, thus pressing the beveled edge of F tight up against that of the flange, *a a*, thereby fastening the spindle, U U, to the fixed parallel plates. It is to remain so while the instrument is being used; U U and all below it then constituting, as it were, a fixed or stationary base, upon which all above it is free to revolve by means of the spindles, T T and *w w*, which may turn in U U, either singly or together, according as their respective clamps are loose or tight, as explained further on.

To remove the upper parts from the parallel plates, loosen the clamp-screw, *q*. Bring the recesses opposite the blocks. Hold back the spring catch, *l*, and lift the upper parts from *m*. When they are so lifted, they are held together by the broad head of the screw which is seen inserted into the foot of the spindle, *w*. This spindle is shown solid, but it is really made hollow in order to reduce the weight of the instrument; and the screw spoken of fits into a plug let into its foot.

Some engineers remove and replace the upper parts of their transit whenever they move it from one stake to the next; but others carry it, all in one, over the shoulder.

We come now to the **Revolving Spindles,** T T and *w w*. **The Outer Spindle,** T T, is cast in one with the **Supporting Plate,** Z Z; so called because it supports the **Graduated Limb,** O O, which is fastened to it by screws (of which two are seen), and of course turns with it, and with the spindle, T T. **The Inner Spindle,** *w w*, has, at its top, a broad flange, by means of which it is fastened by small *screws* (two of which are seen) to the **Vernier Plate,** P P. *The vernier plate* necessarily revolves with this inner spindle, and carries with it all the parts above it, as the **Compass-Box,** C; the standards, V V; the telescope, etc.

To confine the Graduated Limb, O O, to the fixed spindle, U, *tighten the clamp-screw,* H. This presses the split collar, *t t*, tightly against the fixed spindle, U (but not against Z or T). The tongue, *y*, which projects from the collar, is held between the points of two set-screws, of which one, G, is shown, and which move in nuts that are cast in one piece with the supporting plate, Z Z. The latter is thus prevented from revolving when H is clamped, except by the slow-motion, which may still be given to it, and to its graduated limb, by means of the set-screws, G.

To confine the Vernier Plate, P P, the telescope, etc., to prevent them from revolving over the graduated limb, O O, *tighten the clamp-screw, e.* This binds together the two small pieces at its foot, confining between them an edge of the graduated limb. The lower one of these pieces is fastened to one of the two towers, in which works the tangent-screw, *b*, Fig. 53½. The other tower is fixed to the vernier plate. By this tangent-screw we may slightly change the distance apart of the two towers, and thus a slow-motion over the graduated limb can be given to the vernier plate after *e* is clamped tight.

Reviewing briefly, when the transit is in use, the clamp, *q*, always remains fast, and the spindle, U, fixed. By clamping H we prevent Z Z and O O from revolving (except very slightly by means of the set-screws, G). By now clamping *e* we prevent *w w*, P P, the telescope, etc., from revolving horizontally (except slightly by means of the tangent-screw, *b*, Fig. 53½), and the entire instrument is clamped fast. Loosening *e* (H remaining clamped), we release the vernier plate, P P, and allow it, with the telescope, etc., to revolve freely over the still stationary graduated limb, O O. Again, clamping *e* and loosening H, we have T T, Z Z, O O, *w w*, P P, the telescope, etc., free to revolve, *as a whole,* in the fixed spindle, U.

W W is a **Dust-box** surrounding the vernier plate, and protecting it and the graduated limb.

C is the **Compass-box,** which is screwed fast to the vernier plate, P P; and *k k* is the **Needle**; just above *which is seen the glass* cover of the compass-box.

M, Fig. 53¼, is a cross-section of one of the two short **bubble-tubes**; and *r* is one of its capstan-headed adjusting-screws. To their right is seen a curved piece of brass for protecting the bubble-glass. The positions of these two tubes are shown at M M, Fig. 53½.

V V are the **standards** supporting the telescope.

At *p*, Fig. 53½, is one of the two **Verniers** with which the vernier plate is furnished. Both may be read, and their mean taken, when great accuracy is required. Ivory reflectors, *c*, facilitate their reading.

Before the instrument is moved from one station to another, the needle should always be pressed up against the glass cover by means of the milled-head upright screw seen on the vernier plate, just to the right of the nearest standard. Its pivot-point is thus protected from injury.

The Telescope, E O, is usually from 9 to 12 inches long. It is sometimes made to show objects inverted; but more generally upright.

At R, Fig. 53½, is a ring with a clamp (the latter not shown) for holding the telescope in any required position. One end, R, of the axis of the telescope rests in a movable box at the top of the standard. This box may be raised or lowered by means of a screw placed underneath it, and the axis thus adjusted for very slight derangements of the standards. The tangent-screw, whose head, *d*, is seen just below *n n*, moves a vertical arm attached to the clamp-ring at R, and is used for slightly changing the elevation of the telescope in measuring vertical angles, or when using the instrument as a level.

In the vertical arm is a slit, similar to the one seen in the vernier-arm, *l*, of the graduated vertical arc, *g*. When zero of this vernier is placed exactly at 30° on the arc, and the opposite arm placed exactly opposite a small notch on the horizontal brace (not seen in our figs.) of the standard, the two slits will be exactly opposite each other, and may thus be used for laying off offsets, etc., at right angles to the line of sight.

The slide of the object-glass, O, Fig. 53½, is moved backward or forward by a rack and pinion, by means of the milled head, A.

***The slide of* the eye-glass,** E, is sometimes moved in the same way by a milled head, *h;* but often the eye-piece is threaded, and in that case is moved in or out by simply turning it.

The object-slide is protected by a **dust-and-rain-guard,** *a.*

A short brass tube, called a **shade,** is usually furnished with each transit. It is intended to be slid on to the object-end, O, of the telescope, to prevent the glare of the sun upon the object-glass when the sun is low.

The Cross-Hairs.—At B, Fig. 53½, is an outer strengthening ring, see also Figs. 54, carrying four small capstan-screws, *p p, i i.* These screws work in the cross-hair ring, *a,* Figs. 54, which has, stretched across it, two spider-webs, *v* and *h,* usually called the **cross-hairs.** These are much finer than they appear to be, as they are considerably magnified by the eye-glass. The small holes around the heads of the 4 small capstan-screws, *p p, i i,* are for admitting the end of a small steel pin, or lever, for turning them. If first the upper screw be loosened, and then the lower one tightened, the interior ring will be lowered, and the cross-hairs with it; and *vice versa.* The screws, *i i,* at the sides act in the same way for moving the ring sideways. If the telescope is an inverting one, that is, if it makes objects appear inverted, the cross-hairs will appear through the eye-glass to travel in the direction in which they actually move; but when the telescope, as is usual, shows objects *erect,* then the cross-hairs will *appear* to move in the direction opposite to their actual motion, as given by the screws. There is no danger of injuring the *hairs* by turning the capstan-screws, inasmuch as the screws act upon the *ring* only; and, as seen in Figs. 54, do not come in contact with the hairs themselves.

ARTICLE XLIV.

To Adjust a Transit.

When either a level or a transit is purchased it is a good precaution to first screw the object-glass firmly home to its place; and then make a short, continuous scratch upon the ring of the glass, and upon the head of its slide, so as to be sure at any time when at work that the glass is in *the same position, with* regard to the slide. For if, after all the adjustments are completed, the position of the glass should become changed (as it is apt to be if unscrewed, and afterward not screwed up to the same precise spot), the adjustments may thereby become materially deranged if the object-glass is eccentric or not truly ground. Such scratches should be prepared by the maker.

Before making adjustments, as well as while using a transit or level, the eye-glass and object-glass must be so drawn out that there shall be **no parallax**; that is, so that the cross-hairs shall not appear to dance about if the eye is moved a little up or down or sideways. To secure this, take sight at some object, and move the object-glass and eye-glass until the object and the cross-hairs are both seen distinctly; the latter without any of the apparent motion alluded to. After that, the eye-glass must be let alone; and only the object-glass be moved for obtaining distinct vision at different distances.

Make the Adjustments in the Following Order.

1st. To adjust the two short bubble tubes M M, Fig. 53½. By means of the four levelling screws, K, bring the two bubbles to the middles of their tubes. Then turn the upper parts of the instrument half way around. If the bubbles do not remain in place, correct **one-half** of the error by means of the small capstan-headed screws, *r*, of which there are two at one end of each tube; and the other half by the four levelling screws. This operation must be repeated until the bubbles remain at the middles of their tubes while the instrument is being turned entirely around.

2d. See whether the vertical hair is placed truly so in the telescope. To do this, first level up; then take sight at a plumb line, or other vertical object. If the two coincide, the hair is right. But if not, loosen *slightly* only two *adjacent* screws, of the four *p p i i*, Fig. 54, and with a penknife, key, or other light instrument, tap very gently against the sides of the screw heads, until the hair coincides with the plumb line, etc., and then tighten the screws. Two or three trials may be necessary.

As to the horizontal hair, its exact position is not *important; but it is* best to have it near or at the center of the vertical one; and if the instrument is to be used for levelling, or for taking angles of elevation or depression from the horizontal, take care that it is not moved after the adjustments are finished.

3d. To see whether the vertical hair travels vertically while the telescope is being moved up and down.

First, level up; then take sight at some high object, such as the top of a church steeple near by. Clamp, and lower the telescope so as to sight on some low object. If there is no other, drive a stake, or chain-pin, etc., in the line. Unclamp, and revolve the upper parts of the instrument half way around. Clamp, and sight again at the high point. Lower the telescope again to the low point. If the hair still strikes this last it is in order. If not, the standards V V have been deranged, and the instrument must be sent to the maker to be rectified, unless it be provided with an adjusting block and screw under one end of the axis of the telescope, by means of which slight derangement of standards may be counteracted. **One-quarter** of the error must then be corrected by this; and the trial be repeated *de novo;* resetting the stake or chain-pin at each trial.

4th. To adjust the line of collimation, so that the vertical hair shall strike objects in the same straight line on both sides of the instrument, when the telescope is revolved vertically for taking both back and foresights.

Placing the instrument firmly at *a*, Fig. 55, level up, and take sight at any convenient object, *b*, as a chain-pin, stake, etc., distant 100 feet or more. Clamp, and revolving the telescope vertically, observe some other object, as *c*, where the vertical hair then strikes; or better, drive a chain-pin, *c*, in the line. It is not necessary that the distances *a b*, *a c*, be equal; the longer they are the better. Unclamp, and turn the upper parts of the instrument half around horizontally, until the vertical hair again strikes *b*. Clamp, and again revolve the telescope vertically. If the hair now strikes *c*, this adjustment is in order, and *c* is really at o. But if it does *not, observe where it does* strike, say at *m*, and place a pin

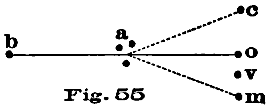


Fig. 55

there also. Measure *m c*, and at **one-fourth** of it, as at *v*, place another pin. Then by the two horizontal screws, *i i*, Fig. 54, move the vertical hair until it strikes *v*, remembering that the hair must be moved in the opposite direction from what *appears* to be the right one, unless the telescope is an inverting one, which is now rarely the case.

The trials must be repeated until the adjustment is perfect.

The foregoing are all the adjustments needed, unless the transit is required for levelling, in which case the following one must be attended to.

5th. To adjust the long bubble tube, F F, Fig. 53½, beneath the telescope, so that when level, it shall be parallel with the line of sight, or of collimation.

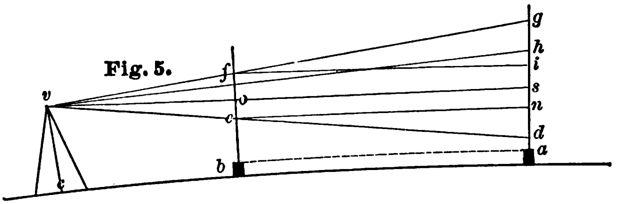


Fig. 5.

Drive two pegs, *a* and *b*, Fig. 56, with their tops at precisely the same level (see Rem. p. 119) and at least about 100 feet apart; 300 or more will be better. Plant the transit firmly, in range with them, as at *c*, making *b c* an aliquot part of *a b*, and as short as will permit focusing on a rod at *b*. The transit need not be leveled. Suppose the line of sight to cut *e* and *d*. Take the readings *b e* and *a d*. Their difference is $b\,e - a\,d = a\,n - a\,d = d\,n$; and $a\,b : a\,c :: d\,n : d\,s$; *s* being the height of the target at *a* when the readings (*a s*, *b o*) on the two stakes are equal. $a\,s = a\,d + d\,s = a\,d + \frac{d\,n \times a\,c}{a\,b}$.

If the reading on *a exceeds* that on *b* (as when the line of sight is *v f g*) the difference of readings is $= a\,g - b\,f = a\,g - a\,i = g\,i$; and $a\,s = a\,g - g\,s = a\,g - \frac{g\,i \times a\,c}{a\,b}$. Sight *to s, bring the* bubble to the center of its tube by means of the two small nuts *n n* at one end of the tube, Fig. 53½, and assume that the telescope and tube are parallel.*

Remark. If no level is at hand, the two stakes, *a b*, Fig. 56, or *m n*, Fig. 57, may be set level by the transit itself, thus: Level the instrument by the 4 levelling screws. Drive one of the stakes, say *m*, at a distance of 100 to 300 feet from the instrument, *o*. Place a target-rod on *m*, and clamp the target tight at any convenient height whatever, as *e v*, at which the horizontal hair may be made to strike it; it being of no importance whether the telescope is level or not. Clamp the telescope by the clamp at R, Fig. 53½, so that it cannot revolve vertically. Then revolve the instrument horizontally a considerable way around; it may be nearly or quite half way; and drive another stake *n*, **at precisely the same distance** from *o* that *m* is; and continue to drive it until the horizontal hair again cuts the target placed on top of it, and still clamped at the same height as when on *m*.

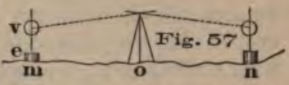


Fig. 57

The tops of the stakes are then on the same level, and ready for the preceding 5th adjustment.

To Replace Broken Cross-Hairs.

These so-called hairs are, in fact, *very fine cobweb;* fine human hair is entirely too coarse.

Take out the tube from the eye end of the telescope; and looking in, notice which side of the cross-hair diaphragm, *a a*, Fig. 54, is turned toward the eye. Then loosen the four screws, *p p*, *i i*, Fig. 54, which hold the diaphragm, so as to let the latter fall out of the telescope. Fasten on new hairs with beeswax, varnish, or gum-arabic water, etc. This requires care. Then, to return the diaphragm to its place, press firmly into one of the screw-holes on its circumference the end of a stick, long enough to reach to where the diaphragm belongs. By this stick, as a handle, insert the diaphragm edgewise into its place; and hold it there until two *opposite* screws are put in place and screwed. Then draw the stick out of the hole in the diaphragm; and with it turn the diaphragm until the same side presents itself to the eye as before; then put in the other two screws.

---

* This neglects a small error due to the curvature of the earth; for a *horizontal* line at *v* is *v h*, tangential to the curved (or "level") surface of still water at *v*, whereas *v s* is tangential to water surface at a point midway between *a* and *b*. Hence if the telescope at *v* points to *s* it will not be parallel to the level bubble-tube. To allow for this, and for the refraction by the air, which *diminishes* the error, raise the target on *a* to a point *h* above *s*. *h s*=.0000000205×square of *a c* in feet; but when *a c* is 650 feet, *h s* is only about one-tenth of an inch and barely covers the apparent thickness of the cross-hair in the telescope.

To Replace a Broken Bubble-Glass.

Detach the bubble tube from the instrument; draw off its sliding ends; push out the broken glass, and the cement which held it. Insert the new glass, with the proper side up (this side is always marked by the maker with a file-mark), wrapping some paper around its ends if it fits loosely. Finally, put a little putty, or melted beeswax, over the ends of the vial, to secure it against moving in its tube.

In purchasing instruments, especially when they are to be used far from a maker, it is advisable to **provide extras** of such parts as may be easily broken or lost; such as glass compass-covers, compass-needles, adjusting-pins, bubble-glasses, magnifiers, etc.

The following is a good form of field-book for the transit and compass combined.

Station.	Distance.	Total Distance.	Course.	Deflection in Degrees.		The right hand page is left blank for Remarks, and Sketches of Topography.
				Left.	Right.	

ARTICLE XLV.

Sines, Tangents, Etc.

The **Complement** of an angle or arc is its difference from 90°. Thus, in Fig. 58, the arc, A B, of 60°, is the complement of B C, which is 30°; and B C is the complement of A B. In like manner, B C is the complement of B C D; B C D that of B C D F; and B C D F that of B C D F A.

The **Supplement** of an angle or arc is its difference from 180°. Thus A B and B C D are supplements to each other: so also A B is the supplement of A B C D E; and B C D is that of B C D F A.

The **Sine** of an angle or arc is a straight line, B W, drawn from either extremity, as B, of the angle or arc, A B, perpendicular to the radius, A X, which joins the other extremity, A, of the arc, and the center, X, of the circle.

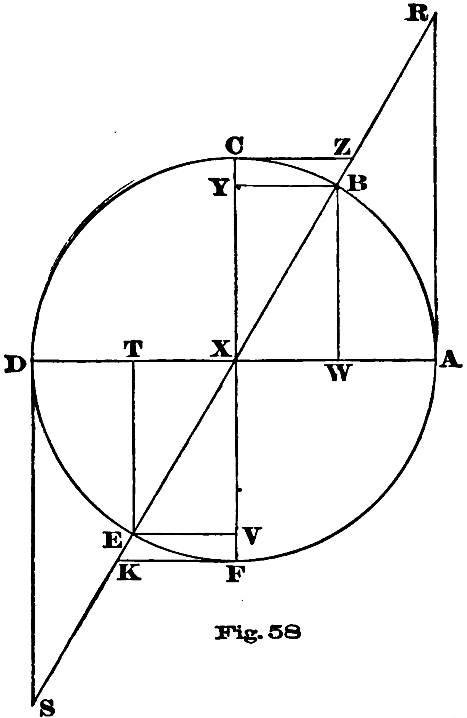


Fig. 58

The **Tangent** is a straight line, R A, touching one extremity, A, of the arc, A B, and limited between that point and its intersection, R, with a secant, X R, which passes through the other extremity, B, of the arc.

The **Secant** is a straight line, X R, drawn from the center, X, of a circle, through one extremity, B, of an arc, A B, to meet the farther extremity, R, of a tangent, R A, *which touches the other* extremity, A, of the arc.

For ***Versed Sines,*** *see page* 169.

The **Cosine** of an angle or arc is the **Sine** of the **Complement** of the arc.

The **Cotangent** is the **Tangent** of the **Complement.**

The **Cosecant** is the **Secant** of the **Complement.**

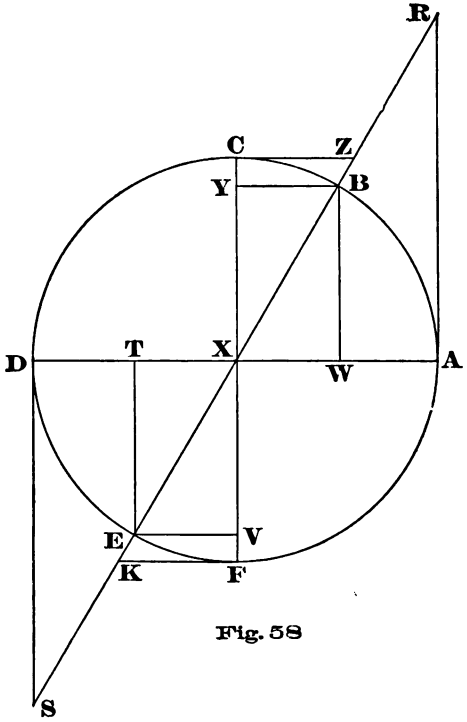


Fig. 58

The Sine, Tangent, Secant, Cosine, Cotangent, and Cosecant of an arc, are respectively = the Sine, Tangent, Secant, Cosine, Cotangent, and Cosecant of the **supplement** of the arc; but the versed sine of an arc is not = the versed sine of the supplement of that arc. See page 169.

Thus, in Fig. 58,

B W (= T E) is the **Sine** of arcs $\overset{60^\circ}{AB}$, $\overset{120^\circ}{BCD}$, $\overset{240^\circ}{ABCDE}$, $\overset{300^\circ}{BCDEFA}$.

A R (= *D S*) is the **Tangent** of the same arcs.

X R (= X S) is the **Secant** of the same arcs.
X W (= X T) is the **Cosine** " " " "
C Z (= F K) is the **Cotangent** " " " "
X Z (= X K) is the **Cosecant** " " " "

Natural Sines, etc., are those for a circle whose radius is 1.

On p. 124 will be found a **Table of Natural Sines, Tangents, Cosines, and Cotangents,** for all arcs from 0° to 90°; and, on p. 123, directions for extending them to all other angles up to 360°; also for finding the secants and cosecants of all angles from 0° to 360°.

Remark.—When, as in Art. XXXIV., an *angle* is to be found from the table by means of its sine, etc., it is important to bear in mind that each sine, etc., in the table, is sine, etc., to *four* different angles, one in each quadrant of the circle, as shown in the remarks on Fig. 58; while the table gives but *one* angle (that in the first quadrant, or between 0° and 90°), for each sine, etc. The four angles thus corresponding to any one sine, etc., are necessarily supplements of each other. The circumstances of the case must determine which of the four is the required angle. Thus, in Fig. 46, x is evidently between 0° and 90°; while in Fig. 47 it is between 90° and 180°.

Remarks on the following Table of Sines, Etc.

The following table does not contain **secants** or **cosecants,** but these may be found thus: **for any angle not exceeding 90°:**

Secant.—Divide 1 by the cosine.
Cosecant.—Divide 1 by the sine.

For versed sines, see Table, p. 170.

For angles exceeding 90°, and less than 180°, take the angle from 180°; if between 180° and 270°, take 180° from the angle; if between 270° and 360°, take the angle from 360°; then, in each case, the *sine, cosine, tangent,* or *cotangent* of the *remainder*, as given by the table, is the *sine, cosine, tangent,* or *cotangent* of the *angle itself;* and the *secant* or *cosecant* of the *remainder*, **found as first directed above,** is the *secant* or *cosecant* of the angle.

For ***Versed Sines,*** *p. 122,* see Table, pages 170 to 192.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

0 Deg. | 0 Deg. | 0 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·0000000	·000000	Infinite.	1·000000	60	21	·0061086	·006108	163·7001	·9999813	39	41	·0119261	·011927	83·84350	·9999289	19
1	·0002909	·000291	3437·746	1·000000	59	22	·0063995	·006399	156·2590	·9999795	38	42	·0122170	·012217	81·84704	·9999254	18
2	·0005818	·000582	1718·873	·9999998	58	23	·0066904	·006690	149·4650	·9999776	37	43	·0125079	·012508	79·94343	·9999218	17
3	·0008727	·000872	1145·915	·9999996	57	24	·0069813	·006981	143·2371	·9999756	36	44	·0127987	·012799	78·12634	·9999181	16
4	·0011636	·001163	859·4363	·9999993	56	25	·0072721	·007272	137·5075	·9999736	35	45	·0130896	·013090	76·39000	·9999143	15
5	·0014544	·001454	687·5488	·9999989	55	26	·0075630	·007563	132·2185	·9999714	34	46	·0133805	·013381	74·72916	·9999105	14
6	·0017453	·001745	572·9572	·9999985	54	27	·0078539	·007854	127·3213	·9999692	33	47	·0136713	·013672	73·13899	·9999065	13
7	·0020362	·002036	491·1060	·9999979	53	28	·0081448	·008145	122·7739	·9999668	32	48	·0139622	·013963	71·61507	·9999025	12
8	·0023271	·002327	429·7175	·9999973	52	29	·0084357	·008436	118·5401	·9999644	31	49	·0142530	·014254	70·15334	·9998984	11
9	·0026180	·002618	381·9709	·9999966	51	30	·0087265	·008726	114·5886	·9999619	30	50	·0145439	·014545	68·75008	·9998942	10
10	·0029089	·002908	343·7737	·9999958	50	31	·0090174	·009017	110·8920	·9999593	29	51	·0148348	·014836	67·40185	·9998900	9
11	·0031998	·003199	312·5213	·9999949	49	32	·0093083	·009308	107·4264	·9999567	28	52	·0151256	·015127	66·10547	·9998856	8
12	·0034907	·003490	286.4777	·9999939	48	33	·0095992	·009599	104·1709	·9999539	27	53	·0154165	·015418	64·85800	·9998812	7
13	·0037815	·003781	264·4408	·9999928	47	34	·0098900	·009890	101·1069	·9999511	26	54	·0157073	·015709	63·65674	·9998766	6
14	·0040724	·004072	245.5519	·9999917	46	35	·0101809	·010181	98·21794	·9999482	25	55	·0159982	·016000	62·49915	·9998720	5
15	·0043633	·004363	229·1816	·9999905	45	36	·0104718	·010472	95·48947	·9999452	24	56	·0162890	·016291	61·38290	·9998673	4
16	·0046542	·004654	214·8576	·9999892	44	37	·0107627	·010763	92·90848	·9999421	23	57	·0165799	·016582	60·30582	·9998625	3
17	·0049451	·004945	202·2187	·9999878	43	38	·0110535	·011054	90·46333	·9999389	22	58	·0168707	·016873	59·26587	·9998577	2
18	·0052360	·005236	190·9841	·9999863	42	39	·0113444	·011345	88·14357	·9999357	21	59	·0171616	·017164	58·26117	·9998527	1
19	·0055268	·005526	180·9322	·9999847	41	40	·0116353	·011636	85·93979	·9999323	20	60	·0174524	·017455	57·28996	·9998477	0
20	·0058177	·005817	171·8854	·9999831	40												
′	Cosine.	Cot n.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 89 | Deg. 89 | Deg. 89

NATURAL SINES AND TANGENTS TO A RADIUS 1

1 Deg. 1 Deg. 1 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·0174524	·017455	57·28996	·9998477	60	21	·0235598	·023566	42·43346	·9997224	39	41	·0293755	·029388	34·02730	·9995684	19
1	·0177432	·017746	56·35059	·9998426	59	22	·0238506	·023857	41·91579	·9997156	38	42	·0296662	·029679	33·69350	·9995599	18
2	·0180341	·018037	55·44151	·9998374	58	23	·0241414	·024148	41·41058	·9997086	37	43	·0299570	·029970	33·36619	·9995512	17
3	·0183249	·018328	54·56130	·9998321	57	24	·0244322	·024439	40·91741	·9997015	36	44	·0302478	·030261	33·04517	·9995424	16
4	·0186158	·018619	53·70858	·9998267	56	25	·0247230	·024730	40·43583	·9996943	35	45	·0305385	·030552	32·73026	·9995336	15
5	·0189066	·018910	52·88211	·9998213	55	26	·0250138	·025021	39·96546	·9996871	34	46	·0308293	·030843	32·42129	·9995247	14
6	·0191974	·019201	52·08067	·9998157	54	27	·0253046	·025312	39·50589	·9996798	33	47	·0311200	·031135	32·11809	·9995157	13
7	·0194883	·019492	51·30315	·9998101	53	28	·0255954	·025603	39·05677	·9996724	32	48	·0314108	·031426	31·82051	·9995066	12
8	·0197791	·019783	50·54850	·9998044	52	29	·0258862	·025894	38·61773	·9996649	31	49	·0317015	·031717	31·52839	·9994974	11
9	·0200699	·020074	49·81572	·9997986	51	30	·0261769	·026185	38·18845	·9996573	30	50	·0319922	·032008	31·24157	·9994881	10
10	·0203608	·020365	49·10388	·9997927	50	31	·0264677	·026477	37·76861	·9996497	29	51	·0322830	·032299	30·95992	·9994788	9
11	·0206516	·020656	48·41208	·9997867	49	32	·0267585	·026768	37·35789	·9996419	28	52	·0325737	·032591	30·68330	·9994693	8
12	·0209424	·020947	47·73950	·9997807	48	33	·0270493	·027059	36·95600	·9996341	27	53	·0328644	·032882	30·41158	·9994598	7
13	·0212332	·021238	47·08534	·9997745	47	34	·0273401	·027350	36·56265	·9996262	26	54	·0331552	·033173	30·14461	·9994502	6
14	·0215241	·021529	46·44886	·9997683	46	35	·0276309	·027641	36·17759	·9996182	25	55	·0334459	·033464	29·88229	·9994405	5
15	·0218149	·021820	45·82935	·9997620	45	36	·0279216	·027932	35·80055	·9996101	24	56	·0337366	·033755	29·62449	·9994308	4
16	·0221057	·022111	45·22614	·9997556	44	37	·0282124	·028223	35·43128	·9996020	23	57	·0340274	·034047	29·37110	·9994209	3
17	·0223965	·022402	44·63859	·9997492	43	38	·0285032	·028514	35·06954	·9995937	22	58	·0343181	·034338	29·12200	·9994110	2
18	·0226873	·022693	44·06611	·9997426	42	39	·0287940	·028805	34·71511	·9995854	21	59	·0346088	·034629	28·87708	·9994009	1
19	·0229781	·022984	43·50812	·9997360	41	40	·0290847	·029097	34·36777	·9995770	20	60	·0348995	·034920	28·63625	·9993908	0
20	·0232690	·023275	42·96407	·9997292	40												
′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′

Deg 88. Deg. 88. Deg. 88.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

2 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·0348995	·034920	28·63625	·9993908	60
1	0351902	·035212	28·39939	·9993806	59
2	·0354809	·035503	28·16642	·9993704	58
3	·0357716	·035794	27·93723	·9993600	57
4	·0360623	·036085	27·71174	·9993495	56
5	·0363530	·036377	27·48985	·9993390	55
6	·0366437	·036668	27·27148	·9993284	54
7	·0369344	·036959	27·05655	·9993177	53
8	·0372251	·037250	26·84498	·9993069	52
9	·0375158	·037542	26·63669	·9992960	51
10	·0378065	·037833	26·43160	·9992851	50
11	·0380971	·028124	26·22963	·9992740	49
12	·0383878	·038416	26·03073	·9992629	48
13	·0386785	·038707	25·83482	·9992517	47
14	·0389692	·038998	25·64183	·9992404	46
15	·0392598	·039290	25·45170	·9992290	45
16	·0395505	·039581	25·26436	·9992176	44
17	·0398411	·039872	25·07975	·9992060	43
18	·0401318	·040164	24·89782	·9991944	42
19	·0404224	·040455	24·71851	·9991827	41
20	·0407131	·040746	24·54175	·9991709	40
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 87.

2 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·0410037	·041038	24·36750	·9991590	39
22	·0412944	·041329	24·19571	·9991470	38
23	·0415850	·041621	24·02632	·9991350	37
24	·0418757	·041912	23·85927	·9991228	36
25	·0421663	·042203	23·69453	·9991106	35
26	·0424569	·042495	23·53205	·9990983	34
27	·0427475	·042786	23·37177	·9990859	33
28	·0430382	·043078	23·21366	·9990734	32
29	·0433288	·043369	23·05767	·9990609	31
30	·0436194	·043660	22·90376	·9990482	30
31	·0439100	·043952	22·75189	·9990355	29
32	·0442006	·044243	22·60201	·9990227	28
33	·0444912	·044535	22·45409	·9990098	27
34	·0447818	·044826	22·30809	·9989968	26
35	·0450724	·045118	22·16398	·9989837	25
36	·0453630	·045409	22·02171	·9989706	24
37	·0456536	·045701	21·88125	·9989573	23
38	·0459442	·045992	21·74256	·9989440	22
39	·0462347	·046284	21·60563	·9989306	21
40	·0465253	·046575	21·47040	·9989171	20
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 87.

2 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·0468159	·046867	21.33685	·9989035	19
42	·0471065	·047158	21·20494	·9988899	18
43	·0473970	·047450	21·07466	·9988761	17
44	·0476876	·047741	20·94596	·9988623	16
45	·0479781	·048033	20·81882	·9988484	15
46	·0482687	·048325	20·69322	·9988344	14
47	·0485592	·048616	20·56911	·9988203	13
48	·0488498	·048908	20·44648	·9988061	12
49	·0491403	·049199	20·32530	·9987919	11
50	·0494308	·049491	20·20555	·9987775	.0
51	·0497214	·049782	20·08719	·9987631	9
52	·0500119	·050074	19·97021	·9987486	8
53	·0503024	·050366	19·85459	·9987340	7
54	·0505929	·050657	19·74029	·9987194	6
55	·0508835	·050949	19·62729	·9987046	5
56	·0511740	·051241	19·51558	·9986898	4
57	·0514645	·051532	19·40513	·9986748	3
58	·0517550	·051824	19·29592	·9986598	2
59	·0520455	·052116	19·18793	·9986447	1
60	·0523360	·052407	19·08113	·9986295	0
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 87.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

3 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·0523360	·052407	19·08113	·9986295	60
1	·0526264	·052699	18·97552	·9986143	59
2	·0529169	·052991	18·87106	·9985989	58
3	·0532074	·053282	18·76775	·9985835	57
4	·0534979	.053574	18·66556	·9985680	56
5	·0537883	·053866	18·56447	·9985524	55
6	·0540788	·054158	18·46447	·9985367	54
7	·0543693	·054449	18·36553	·9985209	53
8	·0546597	·054741	18·26765	·9985050	52
9	·0549502	·055033	18·17080	·9984891	51
10	·0552406	·055325	18·07497	·9984731	50
11	·0555311	·055616	17·98015	·9984570	49
12	·0558215	·055908	17.88631	·9984408	48
13	·0561119	·056200	17·79344	·9984245	47
14	·0564024	·056492	17·70152	·9984081	46
15	·0566928	·056784	17·61055	·9983917	45
16	·0569832	·057075	17·52051	·9983751	44
17	·0572736	·057367	17·43138	·9983585	43
18	·0575640	·057659	17·34315	·9983418	42
19	·0578544	·057951	17·25580	·9983250	41
20	·0581448	·058243	17·16933	·9983082	40
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 86.

3 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·0584352	·058535	17·08372	·9982912	39
22	·0587256	·058827	16·99895	·9982742	38
23	·0590160	·059119	16·91502	·9982570	37
24	·0593064	·059410	16·83191	·9982398	36
25	·0595967	·059702	16·74961	·9982225	35
26	·0598871	·059994	16·66811	·9982052	34
27	·0601775	·060286	16·58739	·9981877	33
28	·0604678	·060578	16·50745	·9981701	32
29	·0607582	·060870	16·42827	·9981525	31
30	·0610485	·061162	16·34985	·9981348	30
31	·0613389	·061454	16·27217	·9981170	29
32	·0616292	·061746	16·19522	·9980991	28
33	·0619196	·062038	16·11899	·9980811	27
34	·0622099	·062330	16·04348	·9980631	26
35	·0625002	·062622	15·96866	·9980450	25
36	·0627905	·062914	15·89454	·9980267	24
37	·0630808	·063206	15·82110	·9980084	23
38	·0633711	·063498	15·74833	·9979900	22
39	·0636614	·063790	15·67623	·9979716	21
40	·0639517	·064082	15·60478	·9979530	20
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 86.

3 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·0642420	·064375	15·53398	·9979343	19
42	·0645323	·064667	15·46381	·9979156	18
43	·0648226	·064959	15·39427	·9978968	17
44	·0651129	·065251	15·32535	·9978779	16
45	·0654031	·065543	15·25705	·9978589	15
46	·0656934	·065835	15·18934	·9978399	14
47	·0659836	·066127	15·12224	·9978207	13
48	·0662739	·066419	15·05572	·9978015	12
49	·0665641	·066712	14·98978	·9977821	11
50	·0668544	·067004	14·92441	·9977627	10
51	·0671446	·067296	14·85961	·9977433	9
52	·0674349	·067588	14·79537	·9977237	8
53	·0677251	·067880	14·73167	·9977040	7
54	·0680153	·068173	14·66852	·9976843	6
55	·0683055	·068465	14·60591	·9976645	5
56	·0685957	·068757	14·54383	·9976445	4
57	·0688859	·069049	14·48227	·9976245	3
58	·0691761	·069342	14·42123	·9976045	2
59	·0694663	·069634	14·36069	·9975843	1
60	·0697565	·069926	14·30066	·9975641	0
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 86.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

2 Deg. 2 Deg. 2 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·0348995	·034920	28·63625	·9993908	60	21	·0410037	·041038	24·36750	·9991590	39	41	·0468159	·046867	21.33685	·9989035	19
1	·0351902	·035212	28·39939	·9993806	59	22	·0412944	·041329	24·19571	·9991470	38	42	·0471065	·047158	21·20494	·9988899	18
2	·0354809	·035503	28·16642	·9993704	58	23	·0415850	·041621	24·02632	·9991350	37	43	·0473970	·047450	21·07466	·9988761	17
3	·0357716	·035794	27·93723	·9993600	57	24	·0418757	·041912	23·85927	·9991228	36	44	·0476876	·047741	20·94596	·9988623	16
4	·0360623	·036085	27·71174	·9993495	56	25	·0421663	·042203	23·69453	·9991106	35	45	·0479781	·048033	20·81882	·9988484	15
5	·0363530	·036377	27·48985	·9993390	55	26	·0424569	·042495	23·53205	·9990983	34	46	·0482687	·048325	20·69322	·9988344	14
6	·0366437	·036668	27·27148	·9993284	54	27	·0427475	·042786	23·37177	·9990859	33	47	·0485592	·048616	20·56911	·9988203	13
7	·0369344	·036959	27·05655	·9993177	53	28	·0430382	·043078	23·21366	·9990734	32	48	·0488498	·048908	20·44648	·9988061	12
8	·0372251	·037250	26·84498	·9993069	52	29	·0433288	·043369	23·05767	·9990609	31	49	·0491403	·049199	20·32530	·9987919	11
9	·0375158	·037542	26·63669	·9992960	51	30	·0436194	·043660	22·90376	·9990482	30	50	·0494308	·049491	20·20555	·9987775	10
10	·0378065	·037833	26·43160	·9992851	50	31	·0439100	·043952	22·75189	·9990355	29	51	·0497214	·049782	20·08719	·9987631	9
11	·0380971	·028124	26·22963	·9992740	49	32	·0442006	·044243	22·60201	·9990227	28	52	·0500119	·050074	19·97021	·9987486	8
12	·0383878	·038416	26·03073	·9992629	48	33	·0444912	·044535	22·45409	·9990098	27	53	·0503024	·050366	19·85459	·9987340	7
13	·0386785	·038707	25·83482	·9992517	47	34	·0447818	·044826	22·30809	·9989968	26	54	·0505929	·050657	19·74029	·9987194	6
14	·0389692	·038998	25·64183	·9992404	46	35	·0450724	·045118	22·16398	·9989837	25	55	·0508835	·050949	19·62729	·9987046	5
15	·0392598	·039290	25·45170	·9992290	45	36	·0453630	·045409	22·02171	·9989706	24	56	·0511740	·051241	19·51558	·9986898	4
16	·0395505	·039581	25·26436	·9992176	44	37	·0456536	·045701	21·88125	·9989573	23	57	·0514645	·051532	19·40513	·9986748	3
17	·0398411	·039872	25·07975	·9992060	43	38	·0459442	·045992	21·74256	·9989440	22	58	·0517550	·051824	19·29592	·9986598	2
18	·0401318	·040164	24·89782	·9991944	42	39	·0462347	·046284	21·60563	·9989306	21	59	·0520455	·052116	19·18793	·9986447	1
19	·0404224	·040455	24·71851	·9991827	41	40	·0465253	·046575	21·47040	·9989171	20	60	·0523360	·052407	19·08113	·9986295	0
20	·0407131	·040746	24·54175	·9991709	40												
′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 87. Deg. 87. Deg. 87.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

3 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·0523360	·052407	19·08113	·9986295	60
1	·0526264	·052699	18·97552	·9986143	59
2	·0529169	·052991	18·87106	·9985989	58
3	·0532074	·053282	18·76775	·9985835	57
4	·0534979	.053574	18·66556	·9985680	56
5	·0537883	·053866	18·56447	·9985524	55
6	·0540788	·054158	18·46447	·9985367	54
7	·0543693	·054449	18·36553	·9985209	53
8	·0546597	·054741	18·26765	·9985050	52
9	·0549502	·055033	18·17080	·9984891	51
10	·0552406	·055325	18·07497	·9984731	50
11	·0555311	·055616	17·98015	·9984570	49
12	·0558215	·055908	17.88631	·9984408	48
13	·0561119	·056200	17·79344	·9984245	47
14	·0564024	·056492	17·70152	·9984081	46
15	·0566928	·056784	17·61055	·9983917	45
16	·0569832	·057075	17·52051	·9983751	44
17	·0572736	·057367	17·43138	·9983585	43
18	·0575640	·057659	17·34315	·9983418	42
19	·0578544	·057951	17·25580	·9983250	41
20	·0581448	·058243	17·16933	·9983082	40
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 86.

3 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·0584352	·058535	17·08372	·9982912	39
22	·0587256	·058827	16·99895	·9982742	38
23	·0590160	·059119	16·91502	·9982570	37
24	·0593064	·059410	16·83191	·9982398	36
25	·0595967	·059702	16·74961	·9982225	35
26	·0598871	·059994	16·66811	·9982052	34
27	·0601775	·060286	16·58739	·9981877	33
28	·0604678	·060578	16·50745	·9981701	32
29	·0607582	·060870	16·42827	·9981525	31
30	·0610485	·061162	16·34985	·9981348	30
31	·0613389	·061454	16·27217	·9981170	29
32	·0616292	·061746	16·19522	·9980991	28
33	·0619196	·062038	16·11899	·9980811	27
34	·0622099	·062330	16·04348	·9980631	26
35	·0625002	·062622	15·96866	·9980450	25
36	·0627905	·062914	15·89454	·9980267	24
37	·0630808	·063206	15·82110	·9980084	23
38	·0633711	·063498	15·74833	·9979900	22
39	·0636614	·063790	15·67623	·9979716	21
40	·0639517	·064082	15·60478	·9979530	20
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 86.

3 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·0642420	·064375	15·53398	·9979343	19
42	·0645323	·064667	15·46381	·9979156	18
43	·0648226	·064959	15·39427	·9978968	17
44	·0651129	·065251	15·32535	·9978779	16
45	·0654031	·065543	15·25705	·9978589	15
46	·0656934	·065835	15·18934	·9978399	14
47	·0659836	·066127	15·12224	·9978207	13
48	·0662739	·066419	15·05572	·9978015	12
49	·0665641	·066712	14·98978	·9977821	11
50	·0668544	·067004	14·92441	·9977627	10
51	·0671446	·067296	14·85961	·9977433	9
52	·0674349	·067588	14·79537	·9977237	8
53	·0677251	·067880	14·73167	·9977040	7
54	·0680153	·068173	14·66852	·9976843	6
55	·0683055	·068465	14·60591	·9976645	5
56	·0685957	·068757	14·54383	·9976445	4
57	·0688859	·069049	14·48227	·9976245	3
58	·0691761	·069342	14·42123	·9976045	2
59	·0694663	·069634	14·36069	·9975843	1
60	·0697565	·069926	14·30066	·9975641	0
′	Cosine.	Cotan.	Tang.	Sine·	′

Deg. 86.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

4 Deg. 4 Deg. 4 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cctang.	Cosine.	′
0	·0697565	·069926	14·30066	·9975641	60	21	·0758489	·076068	13·14612	·9971193	39	41	·0816486	·081922	12·20671	·9966612	19
1	·0700467	·070219	14·24113	·9975437	59	22	·0761390	·076360	13·09575	·9970972	38	42	·0819385	·082215	12·16323	·9966374	18
2	·0703368	·070511	14·18209	·9975233	58	23	·0764290	·076653	13.04576	·9970750	37	43	·0822284	·082507	12·12006	·9966135	17
3	·0706270	·070803	14·12353	·9975028	57	24	·0767190	·076945	12·99616	·9970528	36	44	·0825183	·082800	12·07719	·9965895	16
4	·0709171	·071096	14·06545	·9974822	56	25	·0770091	·077238	12·94692	·9970304	35	45	·0828082	·083093	12·03462	·9965655	15
5	·0712073	·071388	14·00785	·9974615	55	26	·0772991	·077531	12·89805	·9970080	34	46	·0830981	·083386	11·99234	·9965414	14
6	·0714974	·071680	13·95071	·9974408	54	27	·0775891	·077823	12·84955	·9969854	33	47	·0833880	·083679	11·95037	·9965172	13
7	·0717876	·071973	13·89404	·9974199	53	28	·0778791	·078116	12·80141	·9969628	32	48	·0836778	·083972	11·90868	·9964929	12
8	·0720777	·072265	13·83782	·9973990	52	29	·0781691	·078409	12·75363	·9969401	31	49	·0839677	·084265	11·86728	·9964685	11
9	·0723678	·072558	13·78206	·9973780	51	30	·0784591	·078701	12·70620	·9969173	30	50	·0842576	·084558	11·82616	·9964440	10
10	·0726580	·072850	13·72673	·9973569	50	31	·0787491	·078994	12·65912	·9968945	29	51	·0845474	·084851	11·78533	·9964195	9
11	·0729481	·073143	13·67185	·9973357	49	32	·0790391	·079287	12·61239	·9968715	28	52	·0848373	·085144	11·74477	·9963948	8
12	·0732382	·073435	13·61740	·9973145	48	33	·0793290	·079579	12·56599	·9968485	27	53	·0851271	·085437	11·70450	·9963701	7
13	·0735283	·073727	13·56339	·9972931	47	34	·0796190	·079872	12·51994	·9968254	26	54	·0854169	·085730	11·66449	·9963453	6
14	·0738184	·074020	13·50979	·9972717	46	35	·0799090	.080165	12·47422	·9968022	25	55	·0857067	·086023	11·62476	·9963204	5
15	·0741085	·074312	13·45662	·9972502	45	36	·0801989	·080458	12·42883	·9967789	24	56	·0859966	·086316	11·58529	·9962954	4
16	·0743986	·074605	13·40386	·9972286	44	37	·0804889	·080750	12·38376	·9967555	23	57	·0862864	·086609	11·54609	·9962704	3
17	·0746887	·074897	13·35151	·9972069	43	38	·0807788	·081043	12·33902	·9967321	22	58	·0865762	·086902	11·50715	·9962452	2
18	·0749787	·075190	13·29957	·9971851	42	39	·0810687	·081336	12·29460	·9967085	21	59	·0868660	·087195	11·46847	·9962200	1
19	·0752688	·075482	13·24803	·9971633	41	40	·0813587	·081629	12·25050	·9966849	20	60	·0871557	·087488	11·43005	·9961947	0
20	·0755589	·075775	13·19688	·9971413	40												
′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 85. Deg. 85. Deg. 85.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

5 Deg.						5 Deg.						5 Deg.					
′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·0871557	·087488	11·43005	·9961947	60	21	·0932395	·093647	10·67834	·9956437	39	41	·0990303	·099519	10·04828	·9950844	19
1	·0874455	·087781	11·39188	·9961693	59	22	·0935291	·093940	10·64499	·9956165	38	42	·0993197	·099813	10·01871	·9950556	18
2	·0877353	·088074	11·35397	·9961438	58	23	·0938187	·094234	10·61184	·9955892	37	43	·0996092	·100107	9·989305	·9950266	17
3	·0880251	·088368	11·31630	·9961183	57	24	·0941083	·094527	10·57889	·9955620	36	44	·0998986	·100400	9·960072	·9949976	16
4	·0883148	·088661	11·27888	·9960926	56	25	·0943979	·094821	10·54615	·9955345	35	45	·1001881	·100694	9·931008	·9949685	15
5	·0886046	·088954	11·24171	·9960669	55	26	·0946875	·095114	10·51360	·9955070	34	46	·1004775	·100988	9·902112	·9949393	14
6	·0888943	·089247	11·20478	·9960411	54	27	·0949771	·095408	10·48126	·9954794	33	47	·1007669	·101282	9·873382	·9949101	13
7	·0891840	·089540	11·16808	·9960152	53	28	·0952666	·095701	10·44911	·9954517	32	48	·1010563	·101576	9·844816	·9948807	12
8	·0894738	089834	11·13163	·9959892	52	29	·0955562	·095995	10·41715	·9954240	31	49	·1013457	·101870	9·816414	·9948513	11
9	·0897635	·090127	11·09541	·9959631	51	30	·0958458	·096289	10·38539	·9953962	30	50	·1016351	·102164	9·788173	·9948217	10
10	·0900532	·090420	11·05943	·9959370	50	31	·0961353	·096582	10·35382	·9953683	29	51	·1019245	·102458	9·760092	·9947921	9
11	·0903429	·090713	11·02367	·9959107	49	32	·0964248	·096876	10·32244	·9953403	28	52	·1022138	·102752	9·732171	·9947625	8
12	·0906326	·091007	10·98815	·9958844	48	33	·0967144	·097169	10·29125	·9953122	27	53	·1025032	·103046	9·704407	·9947327	7
13	·0909223	·091300	10·95285	·9958580	47	34	·0970039	·097463	10·26024	·9952840	26	54	·1027925	·103339	9·676800	·9947028	6
14	·0912119	·091593	10·91777	·9958315	46	35	·0972934	·097757	10·22942	·9952557	25	55	·1030819	·103634	9·649347	·9946729	5
15	·0915016	·091887	10·88292	·9958049	45	36	·0975829	·098050	10·19878	·9952274	24	56	·1033712	·103928	9·622048	·9946428	4
16	·0917913	·092180	10·84828	·9957783	44	37	·0978724	·098344	10·16833	·9951990	23	57	·1036605	·104222	9·594902	·9946127	3
17	·0920809	·092473	10·81387	·9957515	43	38	·0981619	·098638	10·13805	·9951705	22	58	·1039499	·104516	9·567906	·9945825	2
18	·0923706	·092767	10·77967	·9957247	42	39	·0984514	·098932	10·10795	·9951419	21	59	·1042392	·104810	9·541061	·9945523	1
19	·0926602	·093060	10·74568	·9956978	41	40	·0987408	·099225	10·07803	·9951132	20	60	·1045285	·105104	9·514364	·9945219	0
20	·0929499	·093354	10·71191	·9956708	40												
′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′
				Deg. 84.						Deg. 84.						Deg. 84	

NATURAL SINES AND TANGENTS TO A RADIUS 1.

6 Deg.						6 Deg.						6 Deg.					
′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·1045285	·105104	9·514364	·9945219	60	21	·1106017	·111284	8·985984	·9938648	39	41	·1163818	·117178	8·534017	·9932045	19
1	·1048178	·105398	9·487814	·9944914	59	22	·1108908	·111578	8·962266	·9938326	38	42	·1166707	·117473	8·512594	·9931706	18
2	·1051070	·105692	9·461411	·9944609	58	23	·1111799	·111873	8·938672	·9938003	37	43	·1169596	·117767	8·491277	·9931367	17
3	·1053963	·105986	9·435153	·9944303	57	24	·1114689	·112168	8·915200	·9937679	36	44	·1172485	·118062	8·470065	·9931026	16
4	·1056856	·106280	9·409038	·9943996	56	25	·1117580	·112462	8·891850	·9937355	35	45	·1175374	·118357	8·448957	·9930685	15
5	·1059748	·106575	9·383066	·9943688	55	26	·1120471	·112757	8·868620	·9937029	34	46	·1178263	·118652	8·427953	·9930342	14
6	·1062641	·106869	9·357235	·9943379	54	27	·1123361	·113051	8·845510	·9936703	33	47	·1181151	·118947	8·407051	·9929999	13
7	·1065533	·107163	9·331545	·9943070	53	28	·1126252	·113346	8·822518	·9936375	32	48	·1184040	·119242	8·386251	·9929655	12
8	·1068425	·107457	9·305993	·9942760	52	29	·1129142	·113641	8·799644	·9936047	31	49	·1186928	·119537	8·365553	·9929310	11
9	·1071318	·107751	9·280580	·9942448	51	30	·1132032	·113935	8·776887	·9935719	30	50	·1189816	·119832	8·344955	·9928965	10
10	·1074210	·108046	9·255303	·9942136	50	31	·1134922	·114230	8·754246	·9935389	29	51	·1192704	·120127	8·324457	·9928618	9
11	·1077102	·108340	9·230162	·9941823	49	32	·1137812	·114525	8·731719	·9935058	28	52	·1195593	·120423	8·304058	·9928271	8
12	·1079994	·108634	9·205156	·9941510	48	33	·1140702	·114819	8·709307	·9934727	27	53	·1198481	·120718	8·283757	·9927922	7
13	·1082885	·108929	9·180283	·9941195	47	34	·1143592	·115114	8·687008	·9934395	26	54	·1201368	·121013	8·263554	·9927573	6
14	·1085777	·109223	9·155543	·9940880	46	35	·1146482	·115409	8·664822	·9934062	25	55	·1204256	·121308	8·243418	·9927224	5
15	·1088669	·109517	9·130934	·9940563	45	36	·1149372	·115703	8·642747	·9933728	24	56	·1207144	·121603	8·223488	·9926873	4
16	·1091560	·109812	9·106456	·9940246	44	37	·1152261	·115998	8·620783	·9933393	23	57	·1210031	·121898	8·203523	·9926521	3
17	·1094452	·110106	9·082107	·9939928	43	38	·1155151	·116293	8·598929	·9933057	22	58	·1212919	·122194	8·183704	·9926169	2
18	·1097343	·110401	9·057886	·9939610	42	39	·1158040	·116588	8·577183	·9932721	21	59	·1215806	·122489	8·163978	·9925816	1
19	·1100234	·110695	9·033793	·9939290	41	40	·1160929	·116883	8·555546	·9932384	20	60	·1218693	·122784	8·144346	·9925162	0
20	·1103126	·110989	9·009826	·9938969	40												
′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′
			Deg. 83						Deg. 83.							Deg. 83.	

NATURAL SINES AND TANGENTS TO A RADIUS I.

7 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·1218693	·122784	8·144346	·9925462	60
1	·1221581	·123079	8·124807	·9925107	59
2	·1224468	·123375	8·105359	·9924751	58
3	·1227355	·123670	8·086004	·9924394	57
4	·1230241	·123965	8·066739	·9924037	56
5	·1233128	·124261	8·047564	·9923679	55
6	·1236015	·124556	8·028479	·9923319	54
7	·1238901	·124852	8·009483	·9922959	53
8	·1241788	·125147	7·990575	·9922599	52
9	·1244674	·125442	7·971755	·9922237	51
10	·1247560	·125738	7·953022	·9921874	50
11	·1250446	·126033	7·934375	·9921511	49
12	·1253332	·126329	7·915815	·9921147	48
13	·1256218	·126624	7·897339	·9920782	47
14	·1259104	·126920	7·878948	·9920416	46
15	·1261990	·127216	7·860642	·9920049	45
16	·1264875	·127511	7·842419	·9919682	44
17	·1267761	·127807	7·824279	·9919314	43
18	·1270646	·128103	7·806221	·9918944	42
19	·1273531	·128398	7·788245	·9918574	41
20	·1276416	·128694	7·770350	·9918204	40
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 82.

7 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·1279302	·128990	7·752536	·9917832	39
22	·1282186	·129285	7·734802	·9917459	38
23	·1285071	·129581	7·717148	·9917086	37
24	·1287956	·129877	7·699573	·9916712	36
25	·1290841	·130173	7·682076	·9916337	35
26	·1293725	·130469	7·664658	·9915961	34
27	·1296609	·130764	7·647317	·9915584	33
28	·1299494	·131060	7·630053	·9915206	32
29	·1302378	·131356	7·612865	·9914828	31
30	·1305262	·131652	7·595754	·9914449	30
31	·1308146	·131948	7·578717	·9914069	29
32	·1311030	·132244	7·561756	·9913688	28
33	·1313913	·132540	7·544869	·9913306	27
34	·1316797	·132836	7·528057	·9912923	26
35	·1319681	·133132	7·511317	9912540	25
36	·1322564	·133428	7·494651	·9912155	24
37	·1325447	·133724	7·478057	·9911770	23
38	·1328330	·134020	7·461535	·9911384	22
39	·1331213	·134316	7·445085	·9910997	21
40	·1334096	·134612	7·428706	·9910610	20
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 82.

7 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·1336979	·134909	7·412397	·9910221	19
42	·1339862	·135205	7·396159	·9909832	18
43	·1342744	·135501	7·379990	·9909442	17
44	·1345627	·135797	7·363891	·9909051	16
45	·1348509	·136094	7·347861	·9908659	15
46	·1351392	·136390	7·331898	·9908266	14
47	·1354274	·136686	7·316004	·9907873	13
48	·1357156	·136983	7·300178	·9907478	12
49	·1360038	·137279	7·284418	·9907083	11
50	·1362919	·137575	7·268725	·9906687	10
51	·1365801	·137872	7·253098	·9906290	9
52	·1368683	·138168	7·237537	·9905893	8
53	·1371564	·138465	7·222042	·9905494	7
54	·1374445	·138761	7·206611	·9905095	6
55	·1377327	·139058	7·191245	·9904694	5
56	·1380208	·139354	7·175943	·9904293	4
57	·1383089	·139651	7·160705	·9903891	3
58	·1385970	·139947	7·145530	·9903489	2
59	·1388850	·140244	7·130419	·9903085	1
60	·1391731	·140540	7·115369	·9902681	0
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 82

NATURAL SINES AND TANGENTS TO A RADIUS 1.

8 Deg. 8 Deg. 8 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·1391731	·140540	7·115369	·9902681	60	21	·1452197	·146775	6·813122	·9893994	39	41	·1509733	·152723	6·547767	·9885378	19
1	·1394612	·140837	7·100382	·9902275	59	22	·1455075	·147072	6·799356	·9893572	38	42	·1512608	·153021	6·535029	·9884939	18
2	·1397492	·141134	7·085457	·9901869	58	23	·1457953	·147369	6·785644	·9893148	37	43	·1515484	·153319	6·522339	·9884498	17
3	·1400372	·141430	7·070593	·9901462	57	24	·1460830	·147667	6·771986	·9892723	36	44	·1518359	·153617	6 509698	·9884057	16
4	·1403252	·141727	7·055790	·9901055	56	25	·1463708	·147964	6·758382	·9892298	35	45	·1521234	·153914	6·497104	·9883615	15
5	·1406132	·142024	7·041048	·9900646	55	26	·1466585	·148261	6·744831	·9891872	34	46	·1524109	·154212	6·484558	·9883172	14
6	·1409012	·142321	7·026366	·9900237	54	27	·1469463	·148559	6·731334	·9891445	33	47	·1526984	·154510	6·472059	·9882728	13
7	·1411892	·142617	7·011744	·9899826	53	28	·1472340	·148856	6·717889	·9891017	32	48	·1529858	·154808	6·459607	·9882284	12
8	·1414772	·142914	6·997180	·9899415	52	29	·1475217	·149153	6·704496	·9890588	31	49	·1532733	·155106	6·447201	·9881838	11
9	·1417651	·143211	6·982678	·9899003	51	30	·1478094	·149451	6·691156	·9890159	30	50	·1535607	·155404	6·434842	·9881392	10
10	·1420531	·143508	6·968233	·9898590	50	31	·1480971	·149748	6·677867	·9889728	29	51	·1538482	·155701	6·422530	·9880945	9
11	·1423410	·143805	6·953847	·9898177	49	32	·1483848	·150045	6·664630	·9889297	28	52	·1541356	·155999	6·410263	·9880497	8
12	·1426289	·144102	6·939519	·9897762	48	33	·1486724	·150343	6·651444	·9888865	27	53	·1544230	·156297	6·398042	·9880048	7
13	·1429168	·144399	6·925248	·9897347	47	34	·1489601	·150640	6·638310	·9888432	26	54	·1547104	·156595	6·385866	·9879599	6
14	·1432047	·144696	6·911035	·9896931	46	35	·1492477	·150938	6·625225	·9887998	25	55	·1549978	·156893	6·373735	·9879148	5
15	·1434926	·144993	6·896879	·9896514	45	36	·1495353	·151235	6·612191	·9887564	24	56	·1552851	·157191	6·361650	·9878697	4
16	·1437805	·145290	6·882780	·9896096	44	37	·1498230	·151533	6·599208	·9887128	23	57	·1555725	·157490	6·349609	·9878245	3
17	·1440684	·145587	6·868737	·9895677	43	38	·1501106	·151830	6·586273	·9886692	22	58	·1558598	·157788	6·337612	·9877792	2
18	·1443562	·145884	6·854750	·9895258	42	39	·1503981	·152128	6·573389	·9886255	21	59	·1561472	·158086	6·325660	·9877338	1
19	·1446440	·146181	6·840819	·9894838	41	40	·1506857	·152426	6·560553	·9885817	20	60	·1564345	·158384	6·313751	·9876883	0
20	·1449319	·146478	6·826943	·9894416	40												
′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′	′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 81. Deg. 81 Deg. 81

12

NATURAL SINES AND TANGENTS TO A RADIUS 1.

9 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·1564345	·158384	6·313751	·9876883	60
1	·1567218	·158682	6·301886	·9876428	59
2	·1570091	·158980	6·290065	·9875972	58
3	·1572963	·159279	6·278286	·9875514	57
4	·1575836	·159577	6·266551	·9875057	56
5	·1578708	·159875	6·254858	·9874598	55
6	·1581581	·160174	6·243208	·9874138	54
7	·1584453	·160472	6·231600	·9873678	53
8	·1587325	·160770	6·220034	·9873216	52
9	·1590197	·161069	6·208510	·9872754	51
10	·1593069	·161367	6·197027	·9872291	50
11	·1595940	·161666	6·185586	·9871827	49
12	·1598812	·161964	6·174186	·9871363	48
13	·1601683	·162263	6·162827	·9870897	47
14	·1604555	·162561	6·151508	·9870431	46
15	·1607426	·162860	6·140230	·9869964	45
16	·1610297	·163159	6·128992	·9869496	44
17	·1613167	·163457	6·117794	·9869027	43
18	·1616038	·163756	6·106636	·9868557	42
19	·1618909	·164055	6·095517	·9868087	41
20	·1621779	·164353	6·084438	·9867615	40
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 80

9 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·1624650	·164652	6·073397	·9867143	39
22	·1627520	·164951	6·062396	·9866670	38
23	·1630390	·165250	6·051434	·9866196	37
24	·1633260	·165548	6·040510	·9865722	36
25	·1636129	·165847	6·029624	·9865246	35
26	·1638999	·166146	6·018777	·9864770	34
27	·1641868	·166445	6·007967	·9864293	33
28	·1644738	·166744	5·997195	·9863815	32
29	·1647607	·167043	5·986461	·9863336	31
30	·1650476	·167342	5·975764	·9862856	30
31	·1653345	·167641	5·965104	·9862375	29
32	·1656214	·167940	5·954481	·9861894	28
33	·1659082	·168239	5·943895	·9861412	27
34	·1661951	·168539	5·933345	·9860929	26
35	·1664819	·168838	5·922832	·9860445	25
36	·1667687	·169137	5·912355	·9859960	24
37	·1670556	·169436	5·901913	·9859475	23
38	·1673423	·169735	5·891508	·9858988	22
39	·1676291	·170035	5·881138	·9858501	21
40	·1679159	·170334	5·870804	·9858013	20
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 80.

9 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·1682026	·170633	5·860505	·9857524	19
42	·1684894	·170933	5·850241	·9857035	18
43	·1687761	·171232	5·840011	·9856544	17
44	·1690628	·171532	5·829817	·9856053	16
45	·1693495	·171831	5·819657	·9855561	15
46	·1696362	·172130	5·809531	·9855068	14
47	·1699228	·172430	5·799440	·9854574	13
48	·1702095	·172730	5·789382	·9854079	12
49	·1704961	·173029	5·779358	·9853583	11
50	·1707828	·173329	5·769368	·9853087	10
51	·1710694	·173628	5·759412	·9852590	9
52	·1713560	·173928	5·749488	·9852092	8
53	·1716425	·174228	5·739598	·9851593	7
54	·1719291	·174527	5·729741	·9851093	6
55	·1722156	·174827	5·719917	·9850593	5
56	·1725022	·175127	5·710125	·9850091	4
57	·1727887	·175427	5·700366	·9849589	3
58	·1730752	·175727	5·690639	·9849086	2
59	·1733617	·176027	5·680944	·9848582	1
60	·1736482	·176327	5·671281	·9848078	0
′	Cosine.	Cotan.	Tang.	Sine.	′

Deg. 80.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

10 Deg. 10 Deg. 10 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·1736482	·176327	5·671281	·9848078	60	21	·1796607	·182632	5·475478	·9837286	39	41	·1853808	·188650	5·300801	·9826668	19
1	·1739346	·176626	5·661650	·9847572	59	22	·1799469	·182933	5·466481	·9836763	38	42	·1856666	·188952	5·292350	·9826128	18
2	·1742211	·176926	5·652051	·9847066	58	23	·1802330	·183233	5·457512	·9836239	37	43	·1859524	·189253	5·283925	·9825587	17
3	·1745075	·177226	5·642483	·9846558	57	24	·1805191	·183534	5·448571	·9835715	36	44	·1862382	·189554	5·275525	·9825046	16
4	·1747939	·177527	5·632947	·9846050	56	25	·1808052	·183835	5·439659	·9835189	35	45	·1865240	·189855	5·267151	·9824504	15
5	·1750803	·177827	5 623442	·9845542	55	26	·1810913	·184135	5·430775	·9834663	34	46	·1868098	·190157	5·258803	·9823961	14
6	·1753667	·178127	5·613968	·9845032	54	27	·1813774	·184436	5·421918	·9834136	33	47	·1870956	·190458	5·250480	·9823417	13
7	·1756531	·178427	5·604524	·9844521	53	28	·1816635	·184737	5·413090	·9833608	32	48	·1873813	·190760	5·242183	·9822873	12
8	·1759395	·178727	5·595112	·9844010	52	29	·1819495	·185038	5·404290	·9833079	31	49	·1876670	·191061	5·233911	·9822327	11
9	·1762258	·179027	5·585730	·9843498	51	30	·1822355	·185339	5·395517	·9832549	30	50	·1879528	·191363	5·225664	·9821781	10
10	·1765121	·179327	5·576378	·9842985	50	31	·1825215	·185639	5·386771	·9832019	29	51	·1882385	·191664	5·217442	·9821234	9
11	·1767984	·179628	5·567057	·9842471	49	32	·1828075	·185940	5·378053	·9831487	28	52	·1885241	·191966	5·209245	·9820686	8
12	·1770847	·179928	5·557766	·9841956	48	33	·1830935	·186241	5·369363	·9830955	27	53	·1888098	·192268	5·201073	·9820137	7
13	·1773710	·180228	5·548505	·9841441	47	34	·1833795	·186542	5·360699	·9830422	26	54	·1890954	·192569	5·192926	·9819587	6
14	·1776573	·180529	5·539274	·9840924	46	35	·1836654	·186843	5·352062	·9829888	25	55	·1893811	·192871	5·184803	·9819037	5
15	·1779435	·180829	5·530072	·9840407	45	36	·1839514	·187144	5·343452	·9829353	24	56	·1896667	·193173	5·176705	·9818485	4
16	·1782298	·181129	5·520900	·9839889	44	37	·1842373	·187446	5·334869	·9828818	23	57	·1899523	·193474	5·168631	·9817933	3
17	·1785160	·181430	5·511757	·9839370	43	38	·1845232	·187747	5·326313	·9828282	22	58	·1902379	·193776	5·160581	·9817380	2
18	·1788022	·181730	5·502644	·9838850	42	39	·1848091	·188048	5·317783	·9827744	21	59	·1905234	·194078	5·152555	·9816826	1
19	·1790884	·182031	5·493560	·9838330	41	40	·1850949	·188349	5·309279	·9827206	20	60	·1908090	·194380	5·144554	·9816272	0
20	·1793746	·182331	5·484505	·9837808	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 79. Deg. 79. Deg. 79.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

11 Deg. 11 Deg. 11 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·1908090	·194380	5·144554	·9816272	60	21	·1968018	·200727	4·981881	·9804433	39	41	·2025024	·206786	4·835901	·9792818	19
1	·1910945	·194682	5·136576	·9815716	59	22	·1970870	·201030	4·974381	·9803860	38	42	·2027873	·207090	4·828817	·9792228	18
2	·1913801	·194984	5·128622	·9815160	58	23	·1973722	·201332	4·966903	·9803286	37	43	·2030721	·207393	4·821753	·9791638	17
3	·1916656	·195286	5·120692	·9814603	57	24	·1976573	·201635	4·959447	·9802712	36	44	·2033569	·207696	4·814709	·9791047	16
4	·1919510	·195588	5·112785	·9814045	56	25	·1979425	·201938	4·952012	·9802136	35	45	·2036418	·208000	4·807685	·9790455	15
5	·1922365	·195890	5·104902	·9813486	55	26	·1982276	·202240	4·944599	·9801560	34	46	·2039265	·208303	4·800680	·9789862	14
6	·1925220	·196192	5·097042	·9812927	54	27	·1985127	·202543	4·937206	·9800983	33	47	·2042113	·208607	4·793695	·9789268	13
7	·1928074	·196494	5·089206	·9812366	53	28	·1987978	·202846	4·929835	·9800405	32	48	·2044961	·208910	4·786730	·9788674	12
8	·1930928	·196796	5·081392	·9811805	52	29	·1990829	·203149	4·922485	·9799827	31	49	·2047808	·209214	4·779783	·9788079	11
9	·1933782	·197098	5·073602	·9811243	51	30	·1993679	·203452	4·915157	·9799247	30	50	·2050655	·209518	4·772856	·9787483	10
10	·1936636	·197400	5·065835	·9810680	50	31	·1996530	·203755	4·907849	·9798667	29	51	·2053502	·209821	4·765949	·9786886	9
11	·1939490	·197703	5·058090	·9810116	49	32	·1999380	·204058	4·900562	·9798086	28	52	·2056349	·210125	4·759060	·9786288	8
12	·1942344	·198005	5·050369	·9809552	48	33	·2002230	·204361	4·893295	·9797504	27	53	·2059195	·210429	4·752190	·9785689	7
13	·1945197	·198307	5·042670	·9808986	47	34	·2005080	·204664	4·886049	·9796921	26	54	·2062042	·210733	4·745340	·9785090	6
14	·1948050	·198610	5·034993	·9808420	46	35	·2007930	·204967	4·878824	·9796337	25	55	·2064888	·211036	4·738508	·9784490	5
15	·1950903	·198912	5·027339	·9807853	45	36	·2010779	·205270	4·871620	·9795752	24	56	·2067734	·211340	4·731695	·9783889	4
16	·1953756	·199214	5·019707	·9807285	44	37	·2013629	·205573	4·864435	·9795167	23	57	·2070580	·211644	4·724901	·9783287	3
17	·1956609	·199517	5·012098	·9806716	43	38	·2016478	·205876	4·857271	·9794581	22	58	·2073426	·211948	4·718125	·9782684	2
18	·1959461	·199819	5·004511	·9806147	42	39	·2019327	·206180	4·850128	·9793994	21	59	·2076272	·212252	4·711368	·9782080	1
19	·1962314	·200122	4·996945	·9805576	41	40	·2022176	·206483	4·843004	·9793406	20	60	·2079117	·212556	4·704630	·9781476	0
20	·1965166	·200424	4·989402	·9805005	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 78. Deg. 78. Deg. 78.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

12 Deg. 12 Deg. 12 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·2079117	·212556	4·704630	·9781476	60	21	·2138829	·218949	4·567261	·9768593	39	41	·2195624	·225054	4·443376	·9755985	19
1	·2081962	·212860	4·697910	·9780871	59	22	·2141671	·219254	4·560911	·9767970	38	42	·2198462	·225359	4·437350	·9755345	18
2	·2084807	·213164	4·691208	·9780265	58	23	·2144512	·219559	4·554577	·9767347	37	43	·2201300	·225665	4·431339	·9754706	17
3	·2087652	·213468	4·684524	·9779658	57	24	·2147353	·219864	4·548260	·9766723	36	44	·2204137	·225971	4·425343	·9754065	16
4	·2090497	·213773	4·677859	·9779050	56	25	·2150194	·220169	4·541960	·9766098	35	45	·2206974	·226276	4·419364	·9753423	15
5	·2093341	·214077	4·671212	·9778441	55	26	·2153035	·220474	4·535677	·9765472	34	46	·2209811	·226582	4·413399	·9752781	14
6	·2096186	·214381	4·664583	·9777832	54	27	·2155876	·220779	4·529410	·9764845	33	47	·2212648	·226888	4·407450	·9752138	13
7	·2099030	·214685	4·657972	·9777222	53	28	·2158716	·221084	4·523160	·9764217	32	48	·2215485	·227194	4·401516	·9751494	12
8	·2101874	·214990	4·651378	·9776611	52	29	·2161556	·221389	4·516926	·9763589	31	49	·2218321	·227500	4·395597	·9750849	11
9	·2104718	·215294	4·644803	·9775999	51	30	·2164396	·221694	4·510708	·9762960	30	50	·2221158	·227806	4·389694	·9750203	10
10	·2107561	·215598	4·638245	·9775386	50	31	·2167236	·221999	4·504507	·9762330	29	51	·2223994	·228112	4·383805	·9749556	9
11	·2110405	·215903	4·631705	·9774773	49	32	·2170076	·222305	4·498322	·9761699	28	52	·2226830	·228418	4·377931	·9748909	8
12	·2113248	·216207	4·625183	·9774159	48	33	·2172915	·222610	4·492153	·9761067	27	53	·2229666	·228724	4·372073	·9748261	7
13	·2116091	·216512	4·618678	·9773544	47	34	·2175754	·222915	4·486000	·9760435	26	54	·2232501	·229030	4·366229	·9747612	6
14	·2118934	·216816	4·612190	·9772928	46	35	·2178593	·223221	4·479863	·9759802	25	55	·2235337	·229336	4·360400	·9746962	5
15	·2121777	·217121	4·605720	·9772311	45	36	·2181432	·223526	4·473742	·9759168	24	56	·2238172	·229642	4·354586	·9746311	4
16	·2124619	·217425	4·599268	·9771693	44	37	·2184271	·223831	4·467637	·9758533	23	57	·2241007	229949	4·348786	·9745660	3
17	·2127462	·217730	4·592832	·9771075	43	38	·2187110	·224137	4·461548	·9757897	22	58	·2243842	·230255	4·343001	·9745008	2
18	·2130304	·218035	4·586414	·9770456	42	39	·2189948	·224442	4·455475	·9757260	21	59	·2246676	·230561	4·337231	·9744355	1
19	·2133146	·218340	4·580012	·9769836	41	40	·2192786	·224748	4·449418	·9756623	20	60	·2249511	·230868	4·331475	·9743701	0
20	·2135988	·218644	4·573628	·9769215	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 77. Deg. 77. Deg 77.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

13 Deg. 13 Deg. 13 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·2249511	·230868	4·331475	·9743701	60	21	·2308989	237311	4·213869	·9729777	39	41	·2365555	·243465	4·107356	·9716180	19
1	·2252345	·231174	4·325734	·9743046	59	22	·2311819	·237618	4·208419	·9729105	38	42	·2368381	·243773	4·102164	·9715491	18
2	·2255179	·231481	4·320007	·9742390	58	23	·2314649	·237926	4·202983	·9728432	37	43	·2371207	·244081	4·096985	·9714802	17
3	·2258013	·231787	4·314295	·9741734	57	24	·2317479	·238233	4·197560	·9727759	36	44	·2374033	·244390	4·091817	·9714112	16
4	·2260846	·232094	4·308597	·9741077	56	25	·2320309	·238541	4·192151	·9727084	35	45	·2376859	·244698	4·086662	·9713421	15
5	·2263680	·232400	4·302913	·9740419	55	26	·2323138	·238848	4·186754	·9726409	34	46	·2379684	·245006	4·081519	·9712729	14
6	·2266513	·232707	4·297244	·9739760	54	27	·2325967	·239156	4·181371	·9725733	33	47	·2382510	·245315	4·076389	·9712036	13
7	·2269346	·233014	4·291588	·9739100	53	28	·2328796	·239463	4·176001	·9725056	32	48	·2385335	·245623	4·071270	·9711343	12
8	·2272179	·233320	4·285947	·9738439	52	29	·2331625	·239771	4·170644	·9724378	31	49	·2388159	·245932	4·066164	·9710649	11
9	2275012	·233627	4·280319	·9737778	51	30	·2334454	·240078	4·165299	·9723699	30	50	·2390984	·246240	4·061070	·9709953	10
10	·2277844	·233934	4·274706	·9737116	50	31	·2337282	·240386	4·159968	·9723020	29	51	·2393808	·2465'9	4·055987	·9709258	9
11	2280677	·234241	4·269107	·9736453	49	32	·2340110	·240694	4·154650	·9722339	28	52	·2396633	·246857	4·050917	·9708561	8
12	·2283509	·234547	4·263521	·9735789	48	33	·2342938	·241001	4·149344	·9721658	27	53	·2399457	·247166	4·045859	·9707863	7
13	·2286341	·234854	4·257950	·9735124	47	34	·2345766	·241309	4·144051	·9720976	26	54	·2402280	·247475	4·040812	·9707165	6
14	·2289172	·235161	4·252392	·9734458	46	35	·2348594	·241617	4·138771	·9720294	25	55	·2405104	·247783	4·035777	·9706466	5
15	·2292004	·235468	4·246848	·9733792	45	36	·2351421	·241925	4·133504	·9719610	24	56	·2407927	·248092	4·030755	·9705766	4
16	·2294835	·235775	4·241317	·9733125	44	37	·2354248	·242233	4·128249	·9718926	23	57	·2410751	·248401	4·025744	·9705065	3
17	·2297666	·236082	4·235800	·9732457	43	38	·2357075	·242541	4·123007	·9718240	22	58	·2413574	·248710	4·020744	·9704363	2
18	·2300497	·236390	4·230297	·9731789	42	39	·2359902	·242849	4·117778	·9717554	21	59	·2416396	·249019	4·015757	·9703660	1
19	·2303328	·236697	4·224808	·9731119	41	40	·2362729	·243157	4·112561	·9716867	20	60	·2419219	·249328	4·010780	·9702957	0
20	·2306159	·237004	4·219331	·9730449	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 76. Deg. 76. Deg. 76.

12*

NATURAL SINES AND TANGENTS TO A RADIUS 1.

14 Deg. 14 Deg. 14 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	2419219	·249328	4.010780	·9702957	60	21	·2478445	·255826	3·908901	·9687998	39	41	·2534766	·262034	3·816295	·9673415	19
1	·2422041	·249637	4·005816	·9702253	59	22	·2481263	·256136	3·904171	·9687277	38	42	·2537579	·262345	3·811773	9672678	18
2	·2424863	·249946	4·000863	·9701548	58	23	·2484081	·256446	3·899451	·9686555	37	43	·2540393	·262656	3·807260	·9671939	17
3	·2427685	·250255	3·995922	·9700842	57	24	·2486899	·256756	3·894742	·9685832	36	44	·2543206	·262967	3·802758	·9671200	16
4	·2430507	·250564	3·990992	·9700135	56	25	·2489716	·257066	3·890044	·9685108	35	45	·2546019	·263278	3 798266	·9670459	15
5	·2433329	·250873	3·986073	·9699428	55	26	·2492533	·257376	3·885357	·9684383	34	46	·2548832	·263589	3·793783	9669718	14
6	·2436150	·251182	3·981166	·9698720	54	27	·2495350	·257686	3·880680	·9683658	33	47	·2551645	·263900	3·789310	·9668977	13
7	·2438971	·251491	3·976271	·9698011	53	28	·2498167	·257997	3 876014	·9682931	32	48	·2554458	·264211	3·784848	.9668234	12
8	·2441792	·251801	3·971386	·9697301	52	29	·2500984	·258307	3·871358	·9682204	31	49	·2557270	·264522	3·780395	·9667490	11
9	·2444613	·252110	3·966513	·9696591	51	30	·2503800	·258617	3·866713	·9681476	30	50	·2560082	·264833	3·775951	·9666746	10
10	·2447433	·252420	3·961651	·9695879	50	31	·2506616	·258928	3·862078	·9680748	29	51	·2562894	·265145	3·771518	·9666001	9
11	·2450254	·252729	3·956801	·9695167	49	32	·2509432	·259238	3·857453	·9680018	28	52	·2565705	·265456	3·767094	·9665255	8
12	·2453074	·253038	3·951961	·9694453	48	33	·2512248	·259548	3·852839	·9679288	27	53	·2568517	·265768	3·762680	·9664508	7
13	·2455894	·253348	3·947133	·9693740	47	34	·2515063	·259859	3·848235	·9678557	26	54	·2571328	·266079	3·758276	·9663761	6
14	·2458713	·253658	3·942315	·9693025	46	35	·2517879	·260169	3·843642	·9677825	25	55	·2574139	·266390	3·753881	·9663012	5
15	·2461533	·253967	3·937509	·9692309	45	36	·2520694	·260480	3·839059	·9677092	24	56	·2576950	·266702	3·749496	·9662263	4
16	·2464352	·254277	3·932714	·9691593	44	37	·2523508	·260791	3·834486	·9676358	23	57	·2579760	·267014	3·745120	·9661513	3
17	·2467171	·254587	3·927929	·9690875	43	38	·2526323	·261101	3·829923	·9675624	22	58	·2582570	·267325	3·740754	·9660762	2
18	·2469990	·254896	3·923156	·9690157	42	39	·2529137	·261412	3·825370	·9674888	21	59	·2585381	·267637	3·736398	·9660011	1
19	·2472809	·255206	3·918393	·9689438	41	40	·2531952	·261723	3·820828	·9674152	20	60	·2588190	·267949	3·732050	·9659258	0
20	·2475627	·255516	3·913642	·9688719	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 75. Deg. 75. Deg. 75.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

15 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·2588190	·267949	3·732050	·9659258	60
1	·2591000	·268261	3·727713	·9658505	59
2	·2593810	·268572	3·723384	·9657751	58
3	·2596619	·268884	3·719065	·9656996	57
4	·2599428	·269196	3·714756	·9656240	56
5	2602237	·269508	3·710455	·9655484	55
6	·2605045	·269820	3·706164	·9654726	54
7	·2607853	·270132	3·701883	·9653968	53
8	·2610662	·270444	3·697610	·9653209	52
9	·2613469	·270757	3·693346	·9652449	51
10	·2616277	·271069	3·689092	·9651689	50
11	·2619085	·271381	3·684847	·9650927	49
12	·2621892	·271694	3·680611	·9650165	48
13	·2624699	·272006	3·676384	·9649402	47
14	·2627506	·272318	3·672166	·9648638	46
15	·2630312	·272631	3·667957	·9647873	45
16	·2633118	·272943	3·663757	·9647108	44
17	·2635925	·273256	3·659566	·9646341	43
18	·2638730	·273569	3·655384	·9645574	42
19	·2641536	·273881	3·651211	·9644806	41
20	·2644342	·274194	3·647046	·9644037	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 74.

15 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·2647147	·274507	3·642891	·9643268	39
22	·2649952	·274820	3·638744	·9642497	38
23	·2652757	·275133	3·634606	·9641726	37
24	·2655561	·275445	3·630477	·9640954	36
25	·2658366	·275758	3·626356	·9640181	35
26	·2661170	·276071	3·622244	·9639407	34
27	·2663973	·276385	3·618141	·9638633	33
28	·2666777	·276698	3·614046	·9637858	32
29	·2669581	·277011	3·609960	·9637081	31
30	·2672384	·277324	3·605883	·9636305	30
31	·2675187	·277637	3·601814	·9635527	29
32	·2677989	·277951	3·597754	·9634748	28
33	·2680792	·278264	3·593702	·9633969	27
34	·2683594	·278578	3·589659	·9633189	26
35	·2686396	·278891	3·585624	·9632408	25
36	·2689198	·279205	3·581597	·9631626	24
37	·2692000	·279518	3·577579	·9630843	23
38	·2694801	·279832	3·573569	·9630060	22
39	·2697602	·280145	3·569568	·9629275	21
40	·2700403	·280459	3·565574	·9628490	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 74.

15 De

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·2703204	·280773	3·561590	·9627704	19
42	·2706004	·281087	3·557613	·9626917	18
43	·2708805	·281401	3·553644	·9626130	17
44	·2711605	·281715	3·549684	·9625342	16
45	·2714404	·282029	3·545732	·9624552	15
46	·2717204	·282343	3·541788	·9623762	14
47	·2720003	·282657	3·537852	·9622972	13
48	·2722802	·282971	3·533925	·9622180	12
49	·2725601	·283285	3·530005	·9621387	11
50	·2728400	·283599	3·526093	·9620594	10
51	·2731198	·283914	3·522190	·9619800	9
52	·2733997	·284228	3·518294	·9619005	8
53	·2736794	·284543	3·514407	·9618210	7
54	·2739592	·284857	3·510527	·9617413	6
55	·2742390	·285172	3·506655	·9616616	5
56	·2745187	·285486	3·502791	·9615818	4
57	·2747984	·285801	3·498935	·9615019	3
58	·2750781	·286115	3·495087	·9614219	2
59	·2753577	·286430	3·491247	·9613418	1
60	·2756374	·286745	3·487414	·9612617	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 74.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

16 Deg. 16 Deg. 16 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·2756374	·286745	3·487414	·9612617	60	21	·2815042	·293368	3·408688	·9595600	39	41	·2870819	·299697	3·336699	·9579060	19
1	·2759170	·287060	3·483589	·9611815	59	22	·2817833	·293683	3·405021	·9594781	38	42	·2873605	·300014	3·333173	·9578225	18
2	·2761965	·287375	3·479772	·9611012	58	23	·2820624	·293999	3·401361	·9593961	37	43	·2876391	·300331	3·329654	·9577389	17
3	·2764761	·287690	3·475963	·9610208	57	24	·2823415	·294316	3·397708	·9593140	36	44	·2879177	·300648	3·326141	·9576552	16
4	·2767556	·288005	3·472161	·9609403	56	25	·2826205	·294632	3·394063	·9592318	35	45	·2881963	·300965	3·322636	·9575714	15
5	·2770352	·288320	3·468367	·9608598	55	26	·2828995	·294948	3·390424	·9591496	34	46	·2884748	·301283	3·319137	·9574875	14
6	·2773147	·288635	3·464581	·9607792	54	27	·2831785	·295264	3·386793	·9590672	33	47	·2887533	·301600	3·315645	·9574035	13
7	·2775941	·288950	3·460802	·9606984	53	28	·2834575	·295580	3·383169	·9589848	32	48	·2890318	·301917	3·312159	·9573195	12
8	·2778736	·289265	3·457031	·9606177	52	29	·2837364	·295897	3·379553	·9589023	31	49	·2893103	·302235	3·308681	·9572354	11
9	·2781530	·289580	3·453267	·9605368	51	30	·2840153	·296213	3·375943	·9588197	30	50	·2895887	·302552	3·305209	·9571512	10
10	·2784324	·289896	3·449512	·9604558	50	31	·2842942	·296529	3·372340	·9587371	29	51	·2898671	·302870	3·301743	·9570669	9
11	·2787118	·290211	3·445763	·9603748	49	32	·2845731	·296846	3·368745	·9586543	28	52	·2901455	·303187	3·298285	·9569825	8
12	·2789911	·290526	3·442022	·9602937	48	33	·2848520	·297163	3·365156	·9585715	27	53	·2904239	·303505	3·294833	·9568981	7
13	·2792704	·290842	3·438289	·9602125	47	34	·2851308	·297479	3·361575	·9584886	26	54	·2907022	·303823	3·291387	·9568136	6
14	·2795497	·291157	3·434563	·9601312	46	35	·2854096	·297796	3·358000	·9584056	25	55	·2909805	·304141	3·287948	·9567290	5
15	·2798290	·291473	3·430844	·9600499	45	36	·2856884	·298112	3·354433	·9583226	24	56	·2912588	·304458	3·284516	·9566443	4
16	·2801083	·291789	3·427133	·9599684	44	37	·2859671	·298429	3·350872	·9582394	23	57	·2915371	·304776	3·281090	·9565595	3
17	·2803875	·292104	3·423429	·9598869	43	38	·2862458	·298746	3·347319	·9581562	22	58	·2918153	·305094	3·277671	·9564747	2
18	·2806667	·292420	3·419733	·9598053	42	39	·2865246	·299063	3·343772	·9580729	21	59	·2920935	·305412	3·274258	·9563898	1
19	·2809459	·292736	3·416044	·9597236	41	40	·2868032	·299380	3·340232	·9579895	20	60	·2923717	·305730	3·270852	·9563048	0
20	·2812251	·293052	3·412362	·9596418	40												
	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 73. Deg. 73. Deg. 73.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

17 Deg. 17 Deg. 17 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·2923717	·305730	3·270852	·9563048	60	21	·2982079	·312422	3·200789	·9545009	39	41	·3037559	·318820	3·136563	·9527499	19
1	·2926499	·306048	3·267452	·9562197	59	22	·2984856	·312742	3·197521	·9544141	38	42	·3040331	·319140	3·133414	·9526615	18
2	·2929280	·306367	3·264059	·9561345	58	23	·2987632	·313061	3·194259	·9543273	37	43	·3043102	·319461	3·130270	·9525730	17
3	·2932061	·306685	3·260672	·9560492	57	24	·2990408	·313381	3·191003	·9542403	36	44	·3045872	·319781	3·127131	·9524844	16
4	·2934842	·307003	3·257292	·9559639	56	25	·2993184	·313700	3·187754	·9541533	35	45	·3048643	·320102	3·123999	·9523958	15
5	·2937623	·307321	3·253918	·9558785	55	26	·2995959	·314020	3·184510	·9540662	34	46	·3051413	·320423	3·120872	·9523071	14
6	·2940403	·307640	3·250550	·9557930	54	27	·2998734	·314339	3·181272	·9539790	33	47	·3054183	·320744	3·117750	·9522183	13
7	·2943183	·307958	3·247189	·9557074	53	28	·3001509	·314659	3·178040	·9538917	32	48	·3056953	·321064	3·114635	·9521294	12
8	·2945963	·308277	3·243834	·9556218	52	29	·3004284	·314979	3·174814	·9538044	31	49	·3059723	·321385	3·111525	·9520404	11
9	·2948743	·308595	3·240486	·9555361	51	30	·3007058	·315298	3·171594	·9537170	30	50	·3062492	·321706	3·108421	·9519514	10
10	·2951522	·308914	3·237143	·9554502	50	31	·3009832	·315618	3 168380	·9536294	29	51	·3065261	·322027	3·105322	·9518623	9
11	·2954302	·309233	3·233807	·9553643	49	32	·3012606	·315938	3·165172	·9535418	28	52	·3068030	·322348	3·102229	·9517731	8
12	·2957081	·309551	3·230478	·9552784	48	33	·3015380	·316258	3·161970	·9534542	27	53	·3070798	·322670	3·099141	·9516838	7
13	·2959859	·309870	3·227154	·9551923	47	34	·3018153	·316578	3·158774	·9533664	26	54	·3073566	·322991	3·096059	·9515944	6
14	·2962638	·310189	3·223837	·9551062	46	35	·3020926	·316898	3·155584	·9532786	25	55	·3076334	·323312	3·092983	·9515050	5
15	·2965416	·310508	3·220526	·9550199	45	36	·3023699	·317218	3·152399	·9531907	24	56	·3079102	·323633	3·089912	·9514154	4
16	·2968194	·310827	3·217221	·9549336	44	37	·3026471	·317538	3·149220	·9531027	23	57	·3081869	·323955	3·086846	·9513258	3
17	·2970971	·311146	3·213922	·9548473	43	38	·3029244	·317859	3·146047	·9530146	22	58	·3084636	·324276	3·083786	·9512361	2
18	·2973749	·311465	3·210630	·9547608	42	39	·3032016	·318179	3·142880	·9529264	21	59	·3087403	·324598	3·080732	·9511464	1
19	·2976526	·311784	3·207344	·9546743	41	40	·3034788	·318499	3·139719	·9528382	20	60	·3090170	·324919	3·077683	·9510565	0
20	·2979303	·312103	3·204063	·9545876	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 72. Deg. 72 Deg 72.

NATURAL SINES AND TANGENTS TO A RADIUS 1

18 Deg. 18 Deg. 18 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·3090170	·324919	3·077683	·9510565	60	21	·3148209	·331686	3·014892	·9491511	39	41	·3203374	·338157	2·957205	·9473035	19
1	·3092936	·325241	3·074640	·9509666	59	22	·3150969	·332009	3·011960	·9490595	38	42	·3206130	·338481	2·954372	·9472103	18
2	·3095702	·325563	3·071602	·9508766	58	23	·3153730	·332332	3·009033	·9489678	37	43	·3208885	·338805	2·951545	·9471170	17
3	·3098468	·325884	3·068569	·9507865	57	24	·3156490	·332655	3·006110	·9488760	36	44	·3211640	·339129	2·948722	·9470236	16
4	·3101234	·326206	3·065542	·9506963	56	25	·3159250	·332978	3·003193	·9487842	35	45	·3214395	·339454	2·945905	·9469301	15
5	·3103999	·326528	3·062520	·9506061	55	26	·3162010	·333302	3·000282	·9486922	34	46	·3217149	·339778	2·943092	·9468366	14
6	·3106764	·326850	3·059503	·9505157	54	27	·3164770	·333625	2·997375	·9486002	33	47	·3219903	·340103	2·940284	·9467430	13
7	·3109529	·327172	3·056492	·9504253	53	28	·3167529	·333948	2·994473	·9485081	32	48	·3222657	·340427	2·937480	·9466493	12
8	·3112294	·327494	3·053487	·9503348	52	29	·3170288	·334271	2·991576	·9484159	31	49	·3225411	·340752	2·934682	·9465555	11
9	·3115058	·327816	3·050486	·9502443	51	30	·3173047	·334595	2·988685	·9483237	30	50	·3228164	·341077	2·931888	·9464616	10
10	·3117822	·328138	3·047491	·9501536	50	31	·3175805	·334918	2·985798	·9482313	29	51	·3230917	·341401	2·929099	·9463677	9
11	·3120586	·328461	3·044501	·9500629	49	32	·3178563	·335242	2·982916	·9481389	28	52	·3233670	·341726	2·926315	·9462736	8
12	·3123349	·328783	3·041517	·9499721	48	33	·3181321	·335566	2·980040	·9480464	27	53	·3236422	·342051	2·923535	·9461795	7
13	·3126112	·329105	3·038538	·9498812	47	34	·3184079	·335889	2·977168	·9479538	26	54	·3239174	·342376	2·920761	·9460854	6
14	·3128875	·329428	3·035564	·9497902	46	35	·3186836	·336213	2·974301	·9478612	25	55	·3241926	·342701	2·917990	·9459911	5
15	·3131638	·329750	3·032595	·9496991	45	36	·3189593	·336537	2·971439	·9477684	24	56	·3244678	·343026	2·915225	·9458968	4
16	·3134400	·330073	3·029632	·9496080	44	37	·3192350	·336861	2·968583	·9476756	23	57	·3247429	·343351	2·912464	·9458023	3
17	3137163	·330395	3·026673	·9495168	43	38	·3195106	·337185	2·965731	·9475827	22	58	·3250180	·343677	2·909708	·9457078	2
18	·3139925	·330718	3·023720	·9494255	42	39	·3197863	·337509	2·962884	·9474897	21	59	·3252931	·344002	2·906957	·9456132	1
19	·3142686	·331041	3·020772	·9493341	41	40	·3200619	·337833	2·960042	·9473966	20	60	·3255682	·344327	2·904210	·9455186	0
20	·3145448	·331363	3·017830	·9492426	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 71. Deg. 71. Deg. 71.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

19 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·3255682	·344327	2·904210	·9455186	60
1	·3258432	·344653	2·901468	·9454238	59
2	·3261182	·344978	2·898731	·9453290	58
3	·3263932	·345304	2·895998	·9452341	57
4	·3266681	·345629	2·893270	·9451391	56
5	·3269430	·345955	2·890546	·9450441	55
6	3272179	·346281	2·887827	·9449489	54
7	·3274928	·346606	2·885113	·9448537	53
8	·3277676	·346932	2·882403	·9447584	52
9	·3280424	·347258	2·879697	·9446630	51
10	·3283172	·347584	2·876997	·9445675	50
11	·3285919	·347910	2·874300	·9444720	49
12	·3288666	·348236	2·871608	·9443764	48
13	·3291413	·348563	2·868921	·9442807	47
14	·3294160	·348889	2·866238	·9441849	46
15	·3296906	·349215	2·863560	·9440890	45
16	·3299653	·349542	2·860886	·9439931	44
17	·3302398	·349868	2·858216	·9438971	43
18	·3305144	·350195	2·855551	·9438010	42
19	·3307889	·350521	2·852891	·9437048	41
20	·3310634	·350848	2·850234	·9436085	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 70.

19 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·3313379	·351175	2·847583	·9435122	39
22	·3316123	·351501	2·844935	·9434157	38
23	·3318867	·351828	2·842292	·9433192	37
24	·3321611	·352155	2·839653	·9432227	36
25	·3324355	·352482	2·837019	·9431260	35
26	·3327098	·352809	2·834389	·9430293	34
27	·3329841	·353136	2·831763	·9429324	33
28	·3332584	·353464	2·829142	·9428355	32
29	·3335326	·353791	2·826525	·9427386	31
30	·3338069	·354118	2·823912	·9426415	30
31	·3340810	·354446	2·821304	·9425444	29
32	·3343552	·354773	2·818700	·9424471	28
33	·3346293	·355101	2·816100	·9423498	27
34	·3349034	·355428	2·813504	·9422525	26
35	·3351775	·355756	2·810913	·9421550	25
36	·3354516	·356084	2·808326	·9420575	24
37	·3357256	·356411	2·805743	·9419598	23
38	·3359996	·356739	2·803164	·9418621	22
39	·3362735	·357067	2·800590	·9417644	21
40	·3365475	·357395	2·798019	·9416665	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 70

19 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·3368214	·357723	2·795453	·9415686	19
42	·3370953	·358051	2·792891	·9414705	18
43	·3373691	·358380	2·790333	·9413724	17
44	·3376429	·358708	2·787780	·9412743	16
45	·3379167	·359036	2·785230	·9411760	15
46	·3381905	·359365	2·782685	·9410777	14
47	·3384642	·359693	2·780144	·9409793	13
48	·3387379	·360022	2·777606	·9408808	12
49	·3390116	·360350	2·775073	·9407822	11
50	·3392852	·360679	2·772544	·9406835	10
51	·3395589	·361008	2·770019	·9405848	9
52	·3398325	·361337	2·767499	·9404860	8
53	·3401060	·361666	2·764982	·9403871	7
54	·3403796	·361994	2·762469	·9402881	6
55	·3406531	·362324	2·759960	·9401891	5
56	·3409265	·362653	2·757456	·9400899	4
57	·3412000	·362982	2·754955	·9399907	3
58	·3414734	·363311	2·752458	·9398914	2
59	·3417468	·363640	2·749966	·9397921	1
60	·3420201	·363970	2·747477	·9396926	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 70.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

20 Deg. 20 Deg. 20 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·3420201	·363970	2·747477	·9396926	60	21	·3477540	·370903	2·696118	·9375858	39	41	·3532027	·377536	2·648753	·9355468	19
1	·3422935	·364299	2·744992	·9395931	59	22	·3480267	·371234	2·693714	·9374846	38	42	·3534748	·377868	2·646423	·9354440	18
2	·3425668	·364629	2·742512	·9394935	58	23	·3482994	·371565	2·691314	·9373833	37	43	·3537469	·378201	2·644096	·9353412	17
3	·3428400	·364958	2·740035	·9393938	57	24	·3485720	·371896	2·688919	·9372820	36	44	·3540190	·378533	2·641774	·9352382	16
4	·3431133	·365288	2·737562	·9392940	56	25	·3488447	·372227	2·686526	·9371806	35	45	·3542910	·378866	2·639454	·9351352	15
5	·3433865	·365618	2·735093	·9391942	55	26	·3491173	·372559	2·684138	·9370790	34	46	·3545630	·379198	2·637139	·9350321	14
6	·3436597	·365948	2·732628	·9390943	54	27	·3493898	·372890	2·681753	·9369774	33	47	·3548350	·379531	2·634827	·9349289	13
7	·3439329	·366277	2·730167	·9389943	53	28	·3496624	·373221	2·679372	·9368758	32	48	·3551070	·379864	2·632518	.9348257	12
8	·3442060	·366607	2 727710	·9388942	52	29	·3499349	·373553	2·676995	·9367740	31	49	·3553789	·380197	2·630213	·9347223	11
9	·3444791	·366937	2·725256	·9387940	51	30	·3502074	·373884	2·674621	·9366722	30	50	·3556508	·380530	2·627912	·9346189	10
10	·3447521	·367268	2·722807	·9386938	50	31	·3504798	·374216	2·672251	·9365703	29	51	·3559226	·380863	2·625614	·9345154	9
11	·3450252	·367598	2·720362	·9385934	49	32	·3507523	·374547	2·669885	·9364683	28	52	·3561944	·381196	2·623319	·9344119	8
12	·3452982	·367928	2·717920	·9384930	48	33	.3510246	·374879	2·667522	·9363662	27	53	·3564662	·381529	2·621028	·9343082	7
13	·3455712	·368258	2·715482	·9383925	47	34	·3512970	·375211	2·665163	·9362641	26	54	·3567380	·381862	2·618741	·9342045	6
14	·3458441	·368589	2·713048	·9382920	46	35	·3515693	·375543	2·662808	·9361618	25	55	·3570097	·382196	2·616457	·9341007	5
15	·3461171	·368919	2·710618	·9381913	45	36	·3518416	·375875	2·660456	·9360595	24	56	·3572814	·382529	2·614176	·9339968	4
16	·3463900	·369250	2·708192	·9380906	44	37	·3521139	·376207	2·658108	·9359571	23	57	·3575531	·382863	2·611899	·9338928	3
17	·3466628	·369580	2·705769	·9379898	43	38	·3523862	·376539	2·655764	·9358547	22	58	·3578248	·383196	2·609625	·9337888	2
18	·3469357	·369911	2·703351	·9378889	42	39	·3526584	·376871	2·653423	·9357521	21	59	·3580964	·383530	2·607355	·9336846	1
19	·3472085	·370242	2·700936	·9377880	41	40	·3529306	·377203	2·651086	·9356495	20	60	·3583679	·383864	2·605089	·9335804	0
20	·3474812	·370572	2·698525	·9376869	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 69. Deg. 69. Deg. 69.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

21 Deg. 21 Deg. 21 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·3583679	·383864	2·605089	·9335804	60	21	·3640641	·390889	2·558268	·9313739	39	41	·3694765	·397611	2 515018	·9292401	19
1	·3586395	·384197	2·602825	·9334761	59	22	·3643351	·391224	2·556075	·9312679	38	42	·3697468	·397948	2·512889	·9291326	18
2	·3589110	·384531	2·600565	9333718	58	23	·3646059	·391560	2·553885	·9311619	37	43	·3700170	·398285	2·510762	·9290250	17
3	·3591825	·384865	2·598309	·9332673	57	24	·3648768	·391895	2·551699	·9310558	36	44	·3702872	·398622	2·508639	·9289173	16
4	·3594540	·385199	2·596056	·9331628	56	25	·3651476	·392231	2·549516	·9309496	35	45	·3705574	·398959	2·506519	·9288096	15
5	·3597254	·385533	2·593806	·9330582	55	26	·3654184	·392567	2·547335	·9308434	34	46	·3708276	·399296	2·504403	·9287017	14
6	·3599968	·385867	2·591560	·9329535	54	27	·3656891	·392902	2·545159	·9307370	33	47	·3710977	·399634	2·502289	·9285938	13
7	·3602682	·386202	2·589317	·9328488	53	28	·3659599	·393238	2·542985	·9306306	32	48	·3713678	·399971	2·500178	·9284858	12
8	·3605395	·386536	2·587078	·9327439	52	29	·3662306	·393574	2·540815	·9305241	31	49	·3716379	·400308	2·498070	·9283778	11
9	·3608108	·386870	2·584842	·9326390	51	30	·3665012	·393910	2·538647	·9304176	30	50	·3719079	·400646	2·495966	·9282696	10
10	·3610821	·387205	2·582609	·9325340	50	31	·3667719	·394246	2·536483	·9303109	29	51	·3721780	·400984	2·493864	·9281614	9
11	·3613534	·387539	2·580380	·9324290	49	32	·3670425	·394582	2·534323	·9302042	28	52	·3724479	·401321	2·491766	·9280531	8
12	·3616246	·387874	2·578153	·9323238	48	33	·3673130	·394918	2·532165	·9300974	27	53	·3727179	·401659	2·489670	·9279447	7
13	·3618958	·388209	2·575931	·9322186	47	34	3675836	·395255	2·530011	·9299905	26	54	·3729878	·401997	2·487578	·9278363	6
14	·3621669	·388543	2·573711	·9321133	46	35	·3678541	·395591	2·527859	·9298835	25	55	·3732577	·402335	2·485488	·9277277	5
15	·3624380	·388878	2·571495	·9320079	45	36	·3681246	·395928	2·525711	·9297765	24	56	·3735275	·402673	2·483402	·9276191	4
16	·3627091	·389213	2·569283	·9319024	44	37	·3683950	·396264	2·523566	·9296694	23	57	·3737973	·403011	2·481319	·9275104	3
17	·3629802	·389548	2·567073	·9317969	43	38	·3686654	·396601	2·521424	·9295622	22	58	·3740671	·403349	2·479238	·9274016	2
18	·3632512	·389883	2·564867	·9316912	42	39	·3689358	·396937	2·519286	·9294549	21	59	·3743369	·403687	2·477161	·9272928	1
19	·3635222	·390218	2·562664	·9315855	41	40	·3692061	·397274	2·517150	·9293475	20	60	·3746066	·404026	2·475086	·9271839	0
20	·3637932	·390554	2·560464	·9314797	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 68. Deg. 68. Deg. 68.

NATURA SINES AND TANGENTS TO A RADIUS 1.

22 Deg. 22 Deg. 22 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·3746066	·404026	2·475086	·9271839	60	21	·3802634	·411149	2·432204	·9248782	39	41	·3856377	·417967	2·392531	·9226503	19
1	·3748763	·404364	2·473015	·9270748	59	22	·3805324	·411489	2·430193	·9247676	38	42	·3859060	·418309	2·390576	·9225381	18
2	·3751459	·404703	2·470947	·9269658	58	23	·3808014	·411830	2·428186	·9246568	37	43	·3861744	·418650	2·388625	·9224258	17
3	·3754156	·405041	2·468881	·9268566	57	24	·3810704	·412170	2·426181	·9245460	36	44	·3864427	·418992	2·386675	·9223134	16
4	·3756852	·405380	2·466819	·9267474	56	25	·3813393	·412510	2·4 4180	·9244351	35	45	·3867110	·419334	2·384729	·9222010	15
5	·3759547	·405719	2·464759	·9266380	55	26	·3816082	·412851	2·422181	·9243242	34	46	·3869792	·419676	2·382785	·9220884	14
6	·3762243	·406057	2·462703	·9265286	54	27	·3818770	·413191	2·420185	·9242131	33	47	·3872474	·420019	2·380844	·9219758	13
7	·3764938	·406396	2·460649	·9264192	53	28	·3821459	·413532	2·418191	·9241020	32	48	·3875156	·420361	2·378906	·9218632	12
8	·3767632	·406735	2·458598	·9263096	52	29	·3824147	·413872	2·416201	·9239908	31	49	·3877837	·420703	2·376970	·9217504	11
9	·3770327	·407074	2·456551	·9262000	51	30	·3826834	·414213	2·414213	·9238795	30	50	·3880518	·421046	2·375037	·9216375	10
10	·3773021	·407413	2·454506	·9260902	50	31	·3829522	·414554	2·412228	·9237682	29	51	·3883199	·421388	2·373106	·9215246	9
11	·3775714	·407753	2·452464	·9259805	49	32	·3832209	·414895	2·410246	·9236567	28	52	·3885880	·421731	2·371179	·9214116	8
12	·3778408	·408092	2·450425	·9258706	48	33	·3834895	·415236	2·408267	·9235452	27	53	·3888560	·422073	2·369254	·9212986	7
13	·3781101	·408431	2·448389	·9257606	47	34	·3837582	·415577	2·406290	·9234336	26	54	·3891240	·422416	2·367331	·9211854	6
14	·3783794	·408771	2·446355	·9256506	46	35	·3840268	·415918	2·404316	·9233220	25	55	·3893919	·422759	2·365411	·9210722	5
15	·3786486	·409110	2·444325	·9255405	45	36	·3842953	·416259	2·402345	·9232102	24	56	·3896598	·423102	2·363494	·9209589	4
16	·3789178	·409450	2·442298	·9254303	44	37	·3845639	·416601	2·400377	·9230984	23	57	·3899277	·423445	2·361580	·9208455	3
17	·3791870	·409790	2·440273	·9253201	43	38	·3848324	·416942	2·398411	·9229865	22	58	·3901955	·423788	2·359668	·9207320	2
18	·3794562	·410129	2·438251	·9252097	42	39	·3851008	·417284	2·396449	·9228745	21	59	·3904633	·424131	2·357759	·9206185	1
19	·3797253	·410469	2·436233	·9250993	41	40	·3853693	·417625	2·394488	·9227624	20	60	·3907311	·424474	2·355852	·9205049	0
20	·3799944	·410809	2·434217	·9249888	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 67 Deg. 67. Deg. 67.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

	23 Deg.						23 Deg.						23 Deg.				
′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·3907311	·424474	2·355852	·9205049	60	21	·3963468	·431703	2·316407	·9181009	39	41	·4016814	·438622	2·279865	·9157795	19
1	·3909989	·424818	2·353948	·9203912	59	22	·3966139	·432048	2·314557	·9179855	38	42	·4019478	·438969	2·278063	·9156626	18
2	·3912666	·425161	2·352046	·9202774	58	23	·3968809	·432393	2·312709	·9178701	37	43	·4022141	·439316	2·276264	·9155456	17
3	·3915343	·425505	2·350148	·9201635	57	24	·3971479	·432738	2·310863	·9177546	36	44	·4024804	·439663	2·274467	·9154286	16
4	·3918019	·425848	2·348251	·9200496	56	25	·3974148	·433084	2·309020	·9176391	35	45	·4027467	·440010	2·272672	·9153115	15
5	3920695	·426192	2·346358	·9199356	55	26	·3976818	·433429	2·307180	·9175234	34	46	·4030129	·440357	2·270880	·9151943	14
6	·3923371	·426536	2·344467	·9198215	54	27	·3979486	·433775	2·305342	·9174077	33	47	·4032791	·440705	2·269090	·9150770	13
7	·3926047	·426880	2·342578	·9197073	53	28	·3982155	·434120	2·303506	·9172919	32	48	·4035453	·441052	2·267303	·9149597	12
8	·3928722	·427223	2·340692	·9195931	52	29	·3984823	·434466	2·301673	·9171760	31	49	·4038114	·441400	2·265518	·9148422	11
9	·3931397	·427568	2·338809	·9194788	51	30	·3987491	·434812	2·299842	·9170601	30	50	·4040775	·441747	2·263735	·9147247	10
10	3934071	·427912	2·336928	·9193644	50	31	·3990158	·435158	2·298014	·9169440	29	51	·4043436	·442095	2·261955	·9146072	9
11	·3936745	·428256	2·335050	·9192499	49	32	·3992825	·435504	2·296188	·9168279	28	52	·4046096	·442443	2·260177	·9144895	8
12	·3939419	·428600	2·333174	·9191353	48	33	·3995492	·435850	2·294365	·9167118	27	53	·4048756	·442791	2·258401	·9143718	7
13	·3942093	·428944	2·331301	·9190207	47	34	·3998158	·436196	2·292544	·9165955	26	54	·4051416	·443139	2·256628	·9142540	6
14	·3944766	·429289	2·329431	·9189060	46	35	·4000825	·436542	2·290725	·9164791	25	55	·4054075	·443487	2·254857	·9141361	5
15	·3947439	·429633	2·327563	·9187912	45	36	·4003490	·436889	2·288909	·9163627	24	56	·4056734	·443835	2·253088	·9140181	4
16	·3950111	·429978	2·325697	·9186763	44	37	·4006156	·437235	2·287095	·9162462	23	57	·4059393	·444183	2·251322	·9139001	3
17	·3952783	·430323	2·323834	·9185614	43	38	·4008821	·437582	2·285284	·9161297	22	58	·4062051	·444531	2·249558	·9137819	2
18	·3955455	·430668	2·321974	·9184464	42	39	·4011486	·437928	2·283475	·9160130	21	59	·4064709	·444880	2·247796	·9136637	1
19	·3958127	·431012	2·320116	·9183313	41	40	·4014150	·438275	2·281669	·9158963	20	60	·4067366	·445228	2·246036	·9135455	0
20	·3960798	·4313 7	2·318260	·9182161	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′
				Deg. 66.						Deg. 66.						Deg. 66.	

NATURA SINES AND TANGENTS TO A RADIUS 1.

22 Deg. 22 Deg. 22 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·3746066	·404026	2·475086	·9271839	60	21	·3802634	·411149	2·432204	·9248782	39	41	·3856377	·417967	2·392531	·9226503	19
1	·3748763	·404364	2·473015	·9270748	59	22	·3805324	·411489	2·430193	·9247676	38	42	·3859060	·418309	2·390576	·9225381	18
2	·3751459	·404703	2·470947	·9269658	58	23	·3808014	·411830	2·428186	·9246568	37	43	·3861744	·418650	2·388625	·9224258	17
3	·3754156	·405041	2·468881	·9268566	57	24	·3810704	·412170	2·426181	·9245460	36	44	·3864427	·418992	2·386675	·9223134	16
4	·3756852	·405380	2·466819	·9267474	56	25	·3813393	·412510	2·4 4180	·9244351	35	45	·3867110	·419334	2·384729	·9222010	15
5	·3759547	·405719	2·464759	·9266380	55	26	·3816082	·412851	2·422181	·9243242	34	46	·3869792	·419676	2·382785	·9220884	14
6	·3762243	·406057	2·462703	·9265286	54	27	·3818770	·413191	2·420185	·9242131	33	47	·3872474	·420019	2·380844	·9219758	13
7	·3764938	·406396	2·460649	·9264192	53	28	·3821459	·413532	2·418191	·9241020	32	48	·3875156	·420361	2·378906	·9218632	12
8	·3767632	·406735	2·458598	·9263096	52	29	·3824147	·413872	2·416201	·9239908	31	49	·3877837	·420703	2·376970	·9217504	11
9	·3770327	·407074	2·456551	·9262000	51	30	·3826834	·414213	2·414213	·9238795	30	50	·3880518	·421046	2·375037	·9216375	10
10	·3773021	·407413	2·454506	·9260902	50	31	·3829522	·414554	2·412228	·9237682	29	51	·3883199	·421388	2·373106	·9215246	9
11	·3775714	·407753	2·452464	·9259805	49	32	·3832209	·414895	2·410246	·9236567	28	52	·3885880	·421731	2·371179	·9214116	8
12	·3778408	·408092	2·450425	·9258706	48	33	·3834895	·415236	2·408267	·9235452	27	53	·3888560	·422073	2·369254	·9212986	7
13	·3781101	·408431	2·448389	·9257606	47	34	·3837582	·415577	2·406290	·9234336	26	54	·3891240	·422416	2·367331	·9211854	6
14	·3783794	·408771	2·446355	·9256506	46	35	·3840268	·415918	2·404316	·9233220	25	55	·3893919	·422759	2·365411	·9210722	5
15	·3786486	·409110	2·444325	·9255405	45	36	·3842953	·416259	2·402345	·9232102	24	56	·3896598	·423102	2·363494	·9209589	4
16	·3789178	·409450	2·442298	·9254303	44	37	·3845639	·416601	2·400377	·9230984	23	57	·3899277	·423445	2·361580	·9208455	3
17	·3791870	·409790	2·440273	·9253201	43	38	·3848324	·416942	2·398411	·9229865	22	58	·3901955	·423788	2·359668	·9207320	2
18	·3794562	·410129	2·438251	·9252097	42	39	·3851008	·417284	2·396449	·9228745	21	59	·3904633	·424131	2·357759	·9206185	1
19	·3797253	·410469	2·436233	·9250993	41	40	·3853693	·417625	2·394488	·9227624	20	60	·3907311	·424474	2·355852	·9205049	0
20	·3799944	·410809	2·434217	·9249888	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 67 Deg. 67. Deg. 67.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

23 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·3907311	·424474	2·355852	·9205049	60
1	·3909989	·424818	2·353948	·9203912	59
2	·3912666	·425161	2·352046	·9202774	58
3	·3915343	·425505	2·350148	·9201635	57
4	·3918019	·425848	2·348251	·9200496	56
5	3920695	·426192	2·346358	·9199356	55
6	·3923371	·426536	2·344467	·9198215	54
7	·3926047	·426880	2·342578	·9197073	53
8	·3928722	·427223	2·340692	·9195931	52
9	·3931397	·427568	2·338809	·9194788	51
10	3934071	·427912	2·336928	·9193644	50
11	·3936745	·428256	2·335050	·9192499	49
12	·3939419	·428600	2·333174	·9191353	48
13	·3942093	·428944	2·331301	·9190207	47
14	·3944766	·429289	2·329431	·9189060	46
15	·3947439	·429633	2·327563	·9187912	45
16	·3950111	·429978	2·325697	·9186763	44
17	·3952783	·430323	2·323834	·9185614	43
18	·3955455	·430668	2·321974	·9184464	42
19	·3958127	·431012	2·320116	·9183313	41
20	·3960798	·4313 7	2·318260	·9182161	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 66.

23 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·3963468	·431703	2·316407	·9181009	39
22	·3966139	·432048	2·314557	·9179855	38
23	·3968809	·432393	2·312709	·9178701	37
24	·3971479	·432738	2·310863	·9177546	36
25	·3974148	·433084	2·309020	·9176391	35
26	·3976818	·433429	2·307180	·9175234	34
27	·3979486	·433775	2·305342	·9174077	33
28	·3982155	·434120	2·303506	·9172919	32
29	·3984823	·434466	2·301673	·9171760	31
30	·3987491	·434812	2·299842	·9170601	30
31	·3990158	·435158	2·298014	·9169440	29
32	·3992825	·435504	2·296188	·9168279	28
33	·3995492	·435850	2·294365	·9167118	27
34	·3998158	·436196	2·292544	·9165955	26
35	·4000825	·436542	2·290725	·9164791	25
36	·4003490	·436889	2·288909	·9163627	24
37	·4006156	·437235	2·287095	·9162462	23
38	·4008821	·437582	2·285284	·9161297	22
39	·4011486	·437928	2·283475	·9160130	21
40	·4014150	·438275	2·281669	·9158963	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 66.

23 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·4016814	·438622	2·279865	·9157795	19
42	·4019478	·438969	2·278063	·9156626	18
43	·4022141	·439316	2·276264	·9155456	17
44	·4024804	·439663	2·274467	·9154286	16
45	·4027467	·440010	2·272672	·9153115	15
46	·4030129	·440357	2·270880	·9151943	14
47	·4032791	·440705	2·269090	·9150770	13
48	·4035453	·441052	2·267303	·9149597	12
49	·4038114	·441400	2·265518	·9148422	11
50	·4040775	·441747	2·263735	·9147247	10
51	·4043436	·442095	2·261955	·9146072	9
52	·4046096	·442443	2·260177	·9144895	8
53	·4048756	·442791	2·258401	·9143718	7
54	·4051416	·443139	2·256628	·9142540	6
55	·4054075	·443487	2·254857	·9141361	5
56	·4056734	·443835	2·253088	·9140181	4
57	·4059393	·444183	2·251322	·9139001	3
58	·4062051	·444531	2·249558	·9137819	2
59	·4064709	·444880	2·247796	·9136637	1
60	·4067366	·445228	2·246036	·9135455	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 66.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

24 Deg. 24 Deg. 24 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·4067366	·445228	2·246036	·9135455	60	21	·4123096	·452568	2·209611	·9110438	39	41	·4176028	·459596	2·175822	·9086297	19
1	·4070024	·445577	2·244279	·9134271	59	22	·4125745	·452918	2·207901	·9109238	38	42	·4178671	·459948	2·174155	·9085082	18
2	·4072681	·445926	2·242524	·9133087	58	23	·4128395	·453269	2·206193	·9108038	37	43	·4181313	·460301	2·172491	·9083866	17
3	·4075337	·446274	2·240772	·9131902	57	24	·4131044	·453620	2·204487	·9106837	36	44	·4183956	·460653	2·170828	·9082649	16
4	·4077993	·446623	2·239021	·9130716	56	25	·4133693	·453970	2·202784	·9105635	35	45	·4186597	·461006	2·169167	·9081432	15
5	·4080649	·446972	2·237273	·9129529	55	26	·4136342	·454321	2·201083	·9104432	34	46	·4189239	·461359	2·167509	·9080214	14
6	·4083305	·447321	2·235528	·9128342	54	27	·4138990	·454672	2·199384	·9103228	33	47	·4191880	·461711	2·165852	·9078995	13
7	·4085960	·447670	2·233784	·9127154	53	28	·4141638	·455023	2·197687	·9102024	32	48	·4194521	·462064	2·164198	·9077775	12
8	·4088615	·448020	2·232043	·9125965	52	29	·4144285	·455375	2·195992	·9100819	31	49	·4197161	·462417	2·162546	·9076554	11
9	·4091269	·448369	2·230304	·9124775	51	30	·4146932	·455726	2·194299	·9099613	30	50	·4199801	·462771	2·160895	·9075333	10
10	·4093923	·448718	2·228567	·9123584	50	31	·4149579	·456077	2·192609	·9098406	29	51	·4202441	·463124	2·159247	·9074111	9
11	·4096577	·449068	2·226833	·9122393	49	32	·4152226	·456429	2·190921	·9097199	28	52	·4205080	·463477	2·157601	·9072888	8
12	·4099230	·449417	2·225100	·9121201	48	33	·4154872	·456780	2·189234	·9095990	27	53	·4207719	·463831	2·155957	·9071665	7
13	·4101883	·449767	2·223370	·9120008	47	34	·4157517	·457132	2·187551	·9094781	26	54	·4210358	·464184	2·154315	·9070440	6
14	·4104536	·450117	2·221643	·9118815	46	35	·4160163	·457483	2·185869	·9093572	25	55	·4212996	·464538	2·152675	·9069215	5
15	·4107189	·450467	2·219917	·9117620	45	36	4162808	·457835	2·184189	·9092361	24	56	·4215634	·464891	2·151037	·9067989	4
16	·4109841	·450817	2·218194	·9116425	44	37	·4165453	·458187	2·182511	·9091150	23	57	·4218272	·465245	2·149402	·9066762	3
17	·4112492	·451167	2·216473	·9115229	43	38	·4168097	·458539	2·180836	·9089938	22	58	·4220909	·465599	2·147768	·9065535	2
18	·4115144	·451517	2·214754	·9114033	42	39	·4170741	·458891	2·179163	·9088725	21	59	·4223546	·465953	2·146136	·9064307	1
19	·4117795	·451867	2·213037	·9112835	41	40	·4173385	·459243	2·177492	·9087511	20	60	·4226183	·466307	2·144506	·9063078	0
20	·4120445	·452217	2·211323	·9111637	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 65. Deg. 65. Deg. 65.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

25 Deg. 25 Deg 25 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
* 0	·4226183	·466307	2·144506	·9063078	60	21	·4281467	·473765	2·110747	·9037093	39	41	·4333970	·480909	2·079394	·9012031	19
1	·4228819	·466661	2·142879	·9061848	59	22	·4284095	·474122	2·109161	·9035847	38	42	·4336591	·481267	2·077846	·9010770	18
2	·4231455	·467016	2·141253	·9060618	58	23	·4286723	·474478	2·107577	·9034600	37	43	·4339212	·481625	2·076300	·9009508	17
3	·4231090	467370	2·139630	·9059386	57	24	·4289351	·474834	2·105995	·9033353	36	44	·4341832	·481984	2·074756	·9008246	16
4	·4236725	·467725	2·138008	·9058154	56	25	·4291979	·475191	2·104415	·9032105	35	45	·4344453	·482342	2·073214	·9006982	15
5	·4239360	·468079	2·136389	·9056922	55	26	·4294606	·475548	2·102836	·9030856	34	46	·4347072	·482701	2·071674	·9005718	14
6	·4241994	·468434	2·134771	·9055688	54	27	·4297233	·475904	2·101260	·9029606	33	47	·4349692	·483060	2·070135	·9004453	13
7	·4244628	·468789	2·133155	·9054454	53	28	·4299859	·476261	2·099686	·9028356	32	48	·4352311	·483418	2·068599	·9003188	12
8	·4247262	·469143	2·131542	·9053219	52	29	·4302485	·476618	2·098114	·9027105	31	49	·4354930	·483777	2·067064	·9001921	11
9	·4249895	·469498	2·129930	·9051983	51	30	·4305111	·476975	2·096543	·9025853	30	50	·4357548	·484136	2·065531	·9000654	10
10	·4252528	·469853	2·128321	·9050746	50	31	·4307736	·477332	2·094975	·9024600	29	51	·4360166	·484495	2·064000	·8999386	9
11	·4255161	·470209	2·126713	·9049509	49	32	·4310361	·477689	2·093408	·9023347	28	52	·4362784	·484855	2·062471	·8998117	8
12	·4257793	·470564	2·125108	·9048271	48	33	·4312986	·478047	2·091843	·9022092	27	53	·4365401	·485214	2·060944	·8996848	7
13	·4260425	·470919	2·123504	·9047032	47	34	·4315610	·478404	2·090280	·9020838	26	54	·4368018	·485573	2·059418	·8995578	6
14	·4263056	·471275	2·121903	·9045792	46	35	·4318234	·478762	2·088720	·9019582	25	55	·4370634	·485933	2·057895	·8994307	5
15	·4265687	·471630	2·120303	·9044551	45	36	·4320857	·479119	2·087161	·9018325	24	56	·4373251	·486293	2·056373	·8993035	4
16	·4268318	·471986	2·118705	·9043310	44	37	·4323481	·479477	2·085603	·9017068	23	57	·4375866	·486652	2·054853	·8991763	3
17	·4270949	·472342	2·117110	·9042068	43	38	·4326103	·479835	2·084048	·9015810	22	58	·4378482	·487012	2·053334	·8990489	2
18	·4273579	·472697	2·115516	·9040825	42	39	·4328726	·480193	2·082495	·9014551	21	59	·4381097	·487372	2·051818	·8989215	1
19	·4276208	·473053	2·113924	·9039582	41	40	·4331348	·480551	2·080943	·9013292	20	60	·4383711	·487732	2·050303	·8987940	0
20	·4278838	·473409	2·112334	·9038338	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 64. Deg. 64. Deg. 64.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

26 Deg. 26 Deg. 26 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·4383711	·487732	2·050303	·8987940	60	21	·4438531	·495317	2·018908	·8960994	39	41	·4490591	·502583	1·989720	·8935021	19
1	·4386326	·488092	2·048791	·8986665	59	22	·4441140	·495679	2·017433	·8959703	38	42	·4493190	·502947	1·988278	·8933714	18
2	·4388940	·488453	2·047280	·8985389	58	23	·4443746	·496041	2·015959	·8958411	37	43	·4495789	·503312	1·986838	·8932406	17
3	·4391553	·488813	2·045770	·8984112	57	24	·4446352	·496404	2·014486	·8957118	36	44	·4498387	·503676	1·985400	·8931098	16
4	·4394166	·489173	2·044263	·8982834	56	25	·4448957	·496766	2·013016	·8955824	35	45	·4500984	·504041	1·983963	·8929789	15
5	·4396779	·489534	2 042757	·8981555	55	26	·4451562	·497129	2·011547	·8954529	34	46	·4503582	·504406	1·982528	·8928480	14
6	·4399392	·489894	2·041254	·8980276	54	27	·4454167	·497492	2·010080	·8953234	33	47	·4506179	·504771	1·981095	·8927169	13
7	·4402004	·490255	2·039751	·8978996	53	28	·4456771	·497855	2·008615	8951938	32	48	·4508775	·505136	1·979663	·8925858	12
8	·4404615	·490616	2·038251	·8977715	52	29	·4459375	·498218	2·007151	·8950641	31	49	·4511372	·505501	1·978233	·8924546	11
9	·4407227	·490977	2·036753	·8976433	51	30	·4461978	·498581	2·005689	·8949344	30	50	·4513967	·505866	1·976805	·8923234	10
10	·4409838	·491338	2·035256	·8975151	50	31	·4464581	·498944	2·004229	·8948045	29	51	·4516563	·506232	1·975378	·8921920	9
11	·4412448	·491699	2·033761	·8973868	49	32	·4467184	·499308	2·002771	·8946746	28	52	·4519158	·506597	1·973953	·8920606	8
12	·4415059	·492061	2·032268	·8972584	48	33	·4469786	·499671	2·001314	·8945446	27	53	·4521753	·506963	1·972529	·8919291	7
13	·4417668	·492422	2·030776	·8971299	47	34	·4472388	·500035	1·999859	·8944146	26	54	·4524347	·507329	1·971107	·8917975	6
14	·4420278	·492783	2·029287	·8970014	46	35	·4474990	·500398	1·998405	·8942844	25	55	·4526941	·507694	1·969687	·8916659	5
15	·4422887	·493145	2·027799	·8968727	45	36	·4477591	·500762	1·996953	·8941542	24	56	·4529535	·508060	1·968268	·8915342	4
16	·4425496	·493507	2·026313	·8967440	44	37	·4480192	·501126	1·995503	·8940240	23	57	·4532128	·508426	1·966851	·8914024	3
17	·4428104	·493868	2·024828	·8966153	43	38	·4482792	·501490	1·994055	·8938936	22	58	·4534721	·508792	1·965436	·8912705	2
18	·4430712	·494230	2·023346	·8964864	42	39	·4485392	·501854	1·992608	·8937632	21	59	·4537313	·509159	1·964022	·8911385	1
19	·4433319	·494592	2·021865	·8963575	41	40	·4487992	·502218	1·991163	·8936326	20	60	·4539905	509525	1·962610	·8910065	0
20	·4435927	·494954	2·020386	·8962285	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 63. Deg. 63. Deg. 63.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

27 Deg. 27 Deg. 27 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·4539905	·509525	1·962610	·8910065	60	21	·4594248	517244	1·933323	·8882166	39	41	·4645845	·524640	1·906066	·8855288	19
1	·4542497	·509891	1·961200	8908744	59	22	·4596832	·517612	1·931945	·8880830	38	42	·4648420	·525011	1.904719	·8853936	18
2	·4545088	·510258	1·959791	·8907423	58	23	·4599415	·517981	1·930569	·8879492	37	13	·4650996	·525382	1.903373	·8852584	17
3	·4547679	·510625	1·958383	·8906100	57	24	·4601998	·518350	1·929195	·8878154	36	14	·4653571	·525754	1·902029	·8851230	16
4	·4550269	·510991	1·956978	·8904777	56	25	·4604580	·518719	1·927822	·8876815	35	45	·4656145	·526125	1 900687	·8849876	15
5	·4552859	·511358	1·955573	·8903453	55	26	·4607162	·519089	1·926451	·8875475	34	46	·4658719	·526496	1·899346	·8848522	14
6	·4555449	·511725	1·954171	·8902128	54	27	·4609744	·519458	1·925081	·8874134	33	47	·4661293	·526868	1·898006	·8847166	13
7	·4558038	·512093	1·952770	·8900803	53	28	·4612325	·519827	1·923713	·8872793	32	18	·4663866	·527240	1·896668	·8845810	12
8	·4560627	·512460	1·951371	·8899476	52	29	·4614906	·520197	1·922347	·8871451	31	49	·4666439	·527612	1·895332	·8844453	11
9	·4563216	·512827	1·949973	·8898149	51	30	·4617486	·520567	1·920982	·8870108	30	50	·4669012	·527983	1·893997	·8843095	10
10	·4565804	·513195	1·948577	·8896822	50	31	·4620066	·520936	1·919618	·8868765	29	51	·4671584	·528356	1·892663	·8841736	9
11	·4568392	·513562	1·947182	·8895493	49	32	·4622646	·521306	1·918256	·8867420	28	52	·4674156	·528728	1·891331	·8840377	8
12	·4570979	·513930	1·945789	·8894164	48	33	·4625225	·521676	1·916896	·8866075	27	53	·4676727	·529100	1·890000	·8839017	7
13	·4573566	·514298	1·944398	·8892834	47	34	4627804	·522046	1·915537	·8864730	26	54	·4679298	·529472	1·888671	·8837656	6
14	·4576153	·514665	1·943008	·8891503	46	35	·4630382	·522417	1·914179	·8863383	25	55	·4681869	·529845	1·887343	·8836295	5
15	·4578739	·515033	1·941620	·8890171	45	36	·4632960	·522787	1·912823	·8862036	24	56	·4684439	·530217	1·886017	·8834933	4
16	·4581325	·515401	1·940233	·8888839	44	37	·4635538	·523157	1·911469	·8860688	23	57	·4687009	·530590	1·884692	·8833569	3
17	·4583910	·515770	1·938848	·8887506	43	38	·4638115	·523528	1·910116	·8859339	22	58	·4689578	·530963	1·883369	·8832206	2
18	·4586496	·516138	1·937464	·8886172	42	39	·4640692	·523899	1·908764	8857989	21	59	·4692147	·531336	1·882017	·8830841	1
19	·4589080	·516506	1·936082	·8884838	41	40	·4643269	·524269	1·907414	·8856639	20	60	·4694716	·531709	1·880726	·8829476	0
20	·4591665	·516875	1·934702	·8883503	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 62. Deg. 62. Deg. 62.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

30 Deg. 30 Deg. 30 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·5000000	·577350	1·732050	·8660254	60	21	·5052809	·585524	1·707871	·8629549	39	41	·5102928	·593363	1·685308	·8600007	19
1	·5002519	·577738	1·730887	·8658799	59	22	·5055319	·585914	1·706732	·8628079	38	42	·5105429	·593756	1·684191	·8598523	18
2	·5005037	·578126	1·729726	·8657344	58	23	·5057828	·586305	1·705595	·8626608	37	43	·5107930	·594150	1·683076	·8597037	17
3	·5007556	·578514	1·728565	·8655887	57	24	·5060338	·586696	1·704458	·8625137	36	44	·5110431	·594543	1·681962	·8595551	16
4	·5010073	·578902	1·727406	·8654430	56	25	·5062846	·587087	1·703323	·8623664	35	45	·5112931	·594937	1·680848	·8594064	15
5	·5012591	·579291	1·726247	·8652973	55	26	·5065355	·587478	1·702189	·8622191	34	46	·5115431	·595331	1·679736	·8592576	14
6	·5015107	·579679	1·725090	·8651514	54	27	·5067863	·587870	1·701055	·8620717	33	47	·5117930	·595725	1·678625	·8591088	13
7	·5017624	·580068	1·723934	·8650055	53	28	·5070370	·588261	1·699923	·8619243	32	48	·5120429	·596119	1.677515	·8589599	12
8	·5020140	·580457	1·722779	·8648595	52	29	·5072877	·588653	1·698792	·8617768	31	49	·5122927	·596514	1·676406	·8588109	11
9	·5022655	·580846	1·721626	·8647134	51	30	·5075384	·589045	1·697663	.8616292	30	50	·5125425	·596908	1·675298	·8586619	10
10	·5025170	·581235	1·720473	·8645673	50	31	·5077890	·589436	1·696534	·8614815	29	51	·5127923	·597303	1·674192	·8585127	9
11	·5027685	·581624	1·719322	·8644211	49	32	·5080396	·589828	1·695406	·8613337	28	52	·5130420	·597697	1·673086	·8583635	8
12	·5030199	·582013	1·718172	·8642748	48	33	·5082901	·590221	1·694280	·8611859	27	53	·5132916	·598092	1·671981	·8582143	7
13	·5032713	·582403	1·717023	·8641284	47	34	·5085406	·590613	1·693155	·8610380	26	54	·5135413	·598487	1·670878	·8580649	6
14	·5035227	·582793	1·715875	·8639820	46	35	·5087910	·591005	1·692030	·8608901	25	55	·5137908	·598882	1·669775	·8579155	5
15	·5037740	·583182	1·714728	·8638355	45	36	·5090414	·591398	1·690907	·8607420	24	56	·5140404	·599278	1·668674	·8577660	4
16	·5040252	·583572	1·713582	·8636889	44	37	·5092918	·591791	1·689785	·8605939	23	57	·5142899	·599673	1·667574	·8576164	3
17	·5042765	·583962	1·712438	·8635423	43	38	·5095421	·592183	1·688664	·8604457	22	58	·5145393	·600069	1·666474	·8574668	2
18	·5045276	·584352	1·711294	·8633956	42	39	·5097924	·592576	1·687544	·8602975	21	59	·5147887	·600464	1·665376	·8573171	1
19	·5047788	·584743	1·710152	·8632488	41	40	·5100426	·592969	1·686426	·8601491	20	60	·5150381	·600860	1·664279	·8571673	0
20	·5050298	·585133	1·709011	·8631019	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 59. Deg. 59 Deg. 59.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

31 Deg. 31 Deg. 31 Deg.

'	Sine.	Tang.	Cotang.	Cosine.	'	'	Sine.	Tang.	Cotang.	Cosine.	'	'	Sine.	Tang.	Cotang.	Cosine.	'
0	·5150381	·600860	1·664279	·8571673	60	21	·5202646	·609205	1·641482	·8540051	39	41	·5252241	·617210	1·620192	·8509639	19
1	·5152874	·601256	1·663183	·8570174	59	22	·5205130	·609604	1·640408	·8538538	38	42	·5254717	·617612	1·619138	·8508111	18
2	·5155367	·601652	1·662088	·8568675	58	23	·5207613	·610003	1·639335	·8537023	37	43	·5257191	·618014	1·618085	·8506582	17
3	·5157859	·602049	1·660994	·8567175	57	24	·5210096	·610402	1·638263	·8535508	36	44	·5259665	·618416	1·617033	·8505053	16
4	·5160351	·602445	1·659901	·8565674	56	25	·5212579	.610801	1·637191	·8533992	35	45	·5262139	·618818	1·615982	·8503522	15
5	·5162842	·602841	1·658809	·8564173	55	26	·5215061	·611201	1·636121	·8532475	34	46	·5264613	·619221	1·614932	·8501991	14
6	·5165333	·603238	1·657718	·8562671	54	27	·5217543	·611601	1·635052	·8530958	33	47	·5267085	·619623	1·613882	·8500459	13
7	·5167824	·603635	1·656629	·8561168	53	28	·5220024	·612000	1·633984	·8529440	32	48	·5269558	·620026	1·612834	·8498927	12
8	·5170314	·604032	1·655540	·8559664	52	29	·5222505	·612400	1·632917	·8527921	31	49	·5272030	·620429	1·611787	·8497394	11
9	·5172804	·604429	1·654452	·8558160	51	30	·5224986	·612800	1·631851	·8526402	30	50	·5274502	·620832	1·610741	·8495860	10
10	·5175293	·604826	1·653366	·8556655	50	31	·5227466	·613201	1·630786	·8524881	29	51	·5276973	·621235	1·609696	·8494325	9
11	·5177782	·605224	1·652280	·8555149	49	32	·5229945	·613601	1·629722	·8523360	28	52	·5279443	·621638	1·608652	·8492790	8
12	·5180270	·605621	1·651196	·8553643	48	33	·5232424	·614001	1·628659	·8521839	27	53	·5281914	·622041	1·607609	·8491254	7
13	·5182758	·606019	1·650112	·8552135	47	34	·5234903	·614402	1·627597	·8520316	26	54	·5284383	·622445	1·606567	·8489717	6
14	·5185246	·606417	1·649030	·8550627	46	35	·5237381	·614803	1·626536	·8518793	25	55	·5286853	·622848	1·605526	·8488179	5
15	·5187733	·606814	1·647949	·8549119	45	36	·5239859	·615204	1·625476	·8517269	24	56	·5289322	·623252	1·604485	·8486641	4
16	·5190219	·607213	1·646868	·8547609	44	37	·5242336	·615605	1·624417	·8515745	23	57	·5291790	·623656	1·603446	·8485102	3
17	·5192705	·607611	1·645789	·8546099	43	38	·5244813	·616006	1·623359	·8514219	22	58	·5294258	·624060	1·602408	·8483562	2
18	·5195191	·608009	1·644711	·8544588	42	39	·5247290	·616407	1·622302	·8512693	21	59	·5296726	·624465	1·601370	·8482022	1
19	·5197676	·608408	1·643633	·8543077	41	40	·5249766	·616809	1·621246	·8511167	20	60	·5299193	·624869	1·600334	·8480481	0
20	·5200161	·608806	1·642557	·8541564	40												
'	Cosine.	Cotang.	Tang.	Sine.	'	'	Cosine	Cotang.	Tang.	Sine.	'	'	Cosine.	Cotang.	Tang.	Sine.	'

Deg. 58. Deg. 58. Deg. 58.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

32 Deg. 32 Deg. 32 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·5299193	·624869	1·600634	·8480481	60	21	·5350898	·633395	1·578791	·8447952	39	41	·5399955	·641577	1·558657	·8416679	19
1	·5301659	·625273	1·599299	·8478939	59	22	·5353355	·633803	1·577776	·8446395	38	42	·5402403	·641988	1·557660	·8415108	18
2	·5304125	·625678	1·598264	·8477397	58	23	·5355812	·634211	1·576761	·8444838	37	43	·5404851	·642399	1·556663	·8413536	17
3	·5306591	·626083	1·597231	·8475853	57	24	·5358268	·634619	1·575747	·8443279	36	44	·5407298	·642810	1·555668	·8411963	16
4	·5309057	·626488	1·596198	·8474309	56	25	·5360724	·635027	1·574735	·8441720	35	45	·5409745	·643221	1·554674	·8410390	15
5	·5311521	·626893	1·595167	·8472765	55	26	·5363179	·635435	1·573723	·8440161	34	46	·5412191	·643632	1·553680	·8408816	14
6	·5313986	·627298	1·594136	·8471219	54	27	·5365634	·635844	1·572712	·8438600	33	47	·5414637	·644044	1·552688	·8407241	13
7	·5316450	·627704	1·593107	·8469673	53	28	·5368089	·636252	1·571702	·8437039	32	48	·5417082	·644456	1·551696	·8405666	12
8	·5318913	·628109	1·592078	·8468126	52	29	·5370543	·636661	1·570693	·8435477	31	49	.5419527	·644867	1·550705	·8404090	11
9	·5321376	·628515	1·591050	·8466579	51	30	·5372996	·637070	1·569685	·8433914	30	50	·5421971	·645279	1·549715	·8402513	10
10	·5323839	·628921	1·590023	·8465030	50	31	·5375449	·637479	1·568678	·8432351	29	51	·5424415	·645691	1·548726	·8400936	9
11	·5326301	·629327	1·588997	8463481	49	32	·5377902	·637888	1·567672	·8430787	28	52	·5426859	·646104	1·547738	·8399357	8
12	·5328763	·629733	1·587973	·8461932	48	33	·5380354	·638297	1·566666	·8429222	27	53	·5429302	·646516	1·546751	·8397778	7
13	·5331224	·630139	1·586949	·8460381	47	34	·5382806	·638707	1·565662	·8427657	26	54	·5431744	·646929	1·545764	·8396199	6
14	·5333685	·630546	1·585926	·8458830	46	35	·5385257	·639116	1·564659	·8426091	25	55	·5434187	·647341	1·544779	·8394618	5
15	·5336145	·630953	1·584904	·8457278	45	36	·5387708	·639526	1·563656	·8424524	24	56	·5436628	·647754	1·543794	·8393037	4
16	·5338605	·631359	1·583883	·8455726	44	37	·5390158	·639936	1·562654	·8422956	23	57	·5439069	·648167	1·542810	·8391455	3
17	·5341065	·631766	1·582862	·8454172	43	38	·5392608	·640346	1·561654	·8421388	22	58	·5441510	·648580	1·541828	·8389873	2
18	·5343523	·632173	1·581843	·8452618	42	39	·5395058	·640756	1·560654	·8419819	21	59	·5443951	·648994	1·540846	·8388290	1
19	·5345982	·632581	1·580825	·8451064	41	40	·5397507	·641167	1·559655	·8418249	20	60	·5446390	·649407	1·539865	·8386706	0
20	·5348440	·632988	1·579807	·8449508	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 57. Deg. 57. Deg. 57.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

33 Deg. 33 Deg. 33 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·5446390	·649407	1·539865	·8386706	60	21	·5497520	·658127	1·519463	·8353279	39	41	·5546024	·666496	1·500382	·8321155	19
1	·5448830	·649821	1·538884	·8385121	59	22	·5499950	·658544	1·518501	·8351680	38	42	·5548444	·666917	1·499436	·8319541	18
2	·5451269	·650235	1·537905	·8383536	58	23	·5502379	·658961	1 517540	·8350080	37	43	·5550864	·667337	1·498492	·8317927	17
3	·5453707	·650649	1·536927	·8381950	57	24	·5504807	·659378	1·516579	·8348479	36	44	·5553283	667758	1·497548	·8316312	16
4	·5456145	·651063	1·535949	·8380363	56	25	·5507236	·659796	1·515620	·8346877	35	45	·5555702	·668178	1·496605	·8314696	15
5	·5458583	·651477	1·534972	·8378775	55	26	·5509663	·660213	1·514661	·8345275	34	46	·5558121	·668599	1·495663	·8313080	14
6	·5461020	·651891	1·533996	·8377187	54	27	·5512091	·660631	1·513703	·8343672	33	47	·5560539	·669030	1·494722	·8311463	13
7	·5463456	·652306	1·533021	·8375598	53	28	·5514518	·661049	1·512746	·8342068	32	48	·5562956	·669441	1·493782	·8309845	12
8	·5465892	·652721	1·532047	·8374009	52	29	·5516944	·661467	1·511790	·8340463	31	49	·5565373	·669863	1·492842	·8308226	11
9	·5468328	·653136	1·531074	·8372418	51	30	·5519370	·661885	1·510835	·8338858	30	50	·5567790	·670284	1·491903	8306607	10
10	·5470763	·653551	1·530102	·8370827	50	31	·5521795	·662304	1·509880	·8337252	29	51	·5570206	·670706	1·490965	·8304987	9
11	·5473198	·653966	1·529130	·8369236	49	32	·5524220	·662722	1·508927	·8335646	28	52	·5572621	·671128	1·490028	·8303366	8
12	·5475632	·654381	1·528160	·8367643	48	33	·5526645	·663141	1·507974	·8334038	27	53	·5575036	·671550	1·489092	·8301745	7
13	·5478066	·654797	1·527190	·8366050	47	34	·5529069	·663560	1·507022	·8332430	26	54	·5577451	·671972	1·488157	·8300123	6
14	·5480499	·655212	1·526221	·8364456	46	35	·5531492	·663979	1·506071	8330822	25	55	·5579865	·672394	1·487222	·8298500	5
15	·5482932	·655628	1·525253	·8362862	45	36	·5533915	·664398	1·505121	·8329212	24	56	·5582279	·672816	1·486288	·8296877	4
16	·5485365	·656044	1·524286	·8361266	44	37	·5536338	·664817	1·504171	·8327602	23	57	·5584692	·673239	1·485355	·8295252	3
17	·5487797	·656460	1·523320	·8359670	43	38	·5538760	·665237	1·503222	·8325991	22	58	·5587105	·673662	1·484423	·8293628	2
18	·5490228	·656877	1·522354	·8358074	42	39	·5541182	·665657	1·502275	·8324380	21	59	·5589517	·674085	1·483491	·8292002	1
19	·5492659	·657293	1·521389	·8356476	41	40	·5543603	·666076	1·501328	·8322768	20	60	·5591929	·674508	1·482561	·8290376	0
20	·5495090	·657710	1·520426	·8354878	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 56. Deg. 56. Deg. 56.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

34 Deg. 34 Deg. 34 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·5591929	·674508	1·482561	·8290376	60	21	·5642467	·683433	1·463200	·8256062	39	41	·5690403	·692002	1·445081	·8223096	19
1	·5594340	·674931	1·481631	·8288749	59	22	·5644869	·683860	1·462287	·8254420	38	42	·5692795	·692432	1·444183	·8221440	18
2	·5596751	·675355	1·480702	·8287121	58	23	·5647270	·684287	1·461374	·8252778	37	43	·5695187	·692863	1·443286	·8219784	17
3	·5599162	·675779	1·479773	·8285493	57	24	·5649670	·684714	1·460463	·8251135	36	44	·5697577	·693293	1·442389	·8218127	16
4	·5601572	·676202	1·478846	·8283864	56	25	·5652070	·685141	1·459552	·8249491	35	45	·5699968	·693724	1·441494	·8216469	15
5	·5603981	·676626	1·477919	·8282234	55	26	·5654469	·685569	1·458642	·8247847	34	46	·5702357	·694155	1·440599	·8214811	14
6	·5606390	·677050	1·476993	·8280603	54	27	·5656868	·685996	1·457732	·8246202	33	47	·5704747	·694586	1·439704	·8213152	13
7	·5608798	·677475	1·476068	·8278972	53	28	·5659267	·686424	1·456824	·8244556	32	48	·5707136	·695018	1·438811	·8211492	12
8	·5611206	·677899	1·475144	·8277340	52	29	·5661665	·686852	1·455916	·8242909	31	49	·5709524	·695449	1·437918	·8209832	11
9	·5613614	·678324	1·474221	·8275708	51	30	·5664062	·687281	1·455009	·8241262	30	50	·5711912	·695881	1·437026	·8208170	10
10	·5616021	·678749	1·473298	·8274074	50	31	·5666459	·687709	1·454102	·8239614	29	51	·5714299	·696313	1·436135	·8206509	9
11	·5618428	·679174	1·472376	·8272440	49	32	·5668856	·688137	1·453197	·8237965	28	52	·5716686	·696745	1·435245	·8204846	8
12	·5620834	·679599	1·471455	·8270806	48	33	·5671252	·688566	1·452292	·8236316	27	53	·5719073	·697177	1·434355	·8203183	7
13	·5623239	·680024	1·470535	·8269170	47	34	·5673648	·688995	1·451388	·8234666	26	54	·5721459	·697609	1·433466	·8201519	6
14	·5625645	·680450	1·469615	·8267534	46	35	·5676043	·689424	1·450485	·8233015	25	55	·5723844	.698042	1·432578	·8199854	5
15	·5628049	·680875	1·468696	·8265897	45	36	·5678437	·689853	1·449582	·8231364	24	56	·5726229	·698474	1·431690	·8198189	4
16	·5630453	·681301	1·467778	·8264260	44	37	·5680832	·690283	1·448680	·8229712	23	57	·5728614	·698907	1.430803	·8196523	3
17	·5632857	·681727	1·466861	·8262622	43	38	·5683225	·690712	1·447779	·8228059	22	58	·5730998	·699340	1.429917	·8194856	2
18	·5635260	·682153	1·465945	·8260983	42	39	·5685619	·691142	1·446879	·8226405	21	59	·5733381	·699774	1.429032	·8193189	1
19	·5637663	·682580	1·465029	·8259343	41	40	·5688011	·691572	1·445980	·8224751	20	60	·5735764	·700207	1.428148	·8191520	0
20	·5640066	·683006	1·464114	·8257703	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 55. Deg. 55 Deg. 55

NATURAL SINES AND TANGENTS TO A RADIUS

35 Deg.

′	Sine.	Tang	Cotang.	Cosine.	′
0	·5735764	·700207	1·428148	·8191520	60
1	5738147	·700641	1·427264	·8189852	59
2	5740529	·701074	1·426381	·8188182	58
3	5742911	·701508	1·425498	·8186512	57
4	·5745292	·701943	1·424617	·8184841	56
5	5747672	·702377	1·423736	·8183169	55
6	·5750053	·702811	1·422856	·8181497	54
7	·5752432	·703246	1·421976	·8179824	53
8	·5754811	·703681	1·421097	·8178151	52
9	·5757190	·704116	1·420220	·8176476	51
10	·5759568	·704551	1·419342	·8174801	50
11	·5761946	·704986	1·418466	·8173125	49
12	·5764323	·705422	1·417590	·8171449	48
13	·5766700	·705858	1·416715	·8169772	47
14	·5769076	·706294	1·415840	·8168094	46
15	·5771452	·706730	1·414967	·8166416	45
16	·5773827	·707166	1·414094	·8164736	44
17	·5776202	·707602	1·413222	·8163056	43
18	·5778576	·708039	1·412350	·8161376	42
19	·5780950	·708476	1·411479	·8159695	41
20	·5783323	·708913	1·410609	8158013	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 54.

35 Deg.

′	Sine.	Tang.	Cotang.	Cosine	′
21	·5785696	·709350	1·409740	·8156330	39
22	·5788069	·709787	1·408871	·8154647	38
23	·5790440	·710225	1·408003	·8152963	37
24	·5792812	·710663	1·407136	·8151278	36
25	.5795183	·711100	1·406270	·8149593	35
26	·5797553	·711539	1·405404	·8147906	34
27	·5799923	·711977	1·404539	·8146220	33
28	·5802292	·712415	1·403674	·8144532	32
29	·5804661	·712854	1·402811	·8142844	31
30	·5807030	·713293	1·401948	·8141155	30
31	·5809397	·713732	1·401086	·8139466	29
32	·5811765	·714171	1·400224	·8137775	28
33	·5814132	·714610	1·399363	·8136084	27
34	·5816498	·715050	1·398503	·8134393	26
35	·5818864	·715489	1·397644	·8132701	25
36	·5821230	·715929	1·396785	·8131008	24
37	·5823595	·716369	1·395927	·8129314	23
38	·5825959	·716810	1·395069	·8127620	22
39	·5828323	·717250	1·394213	·8125925	21
40	·5830687	·717691	1·393357	·8124229	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 54.

35 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·5833050	·718131	1·392501	·8122532	19
42	·5835412	·718572	1·391647	·8120835	18
43	·5837774	·719014	1·390793	·8119137	17
44	·5840136	·719455	1·389940	·8117439	16
45	·5842497	·719897	1·389087	·8115740	15
46	·5844857	·720338	1·388235	·8114040	14
47	·5847217	·720780	1·387384	·8112339	13
48	·5849577	·721222	1·386534	·8110638	12
49	·5851936	·721665	1·385684	·8108936	11
50	·5854294	·722107	1·384835	·8107234	10
51	·5856652	·722550	1·383986	·8105530	9
52	·5859010	·722993	1·383139	·8103826	8
53	·5861367	·723436	1·382292	·8102122	7
54	·5863724	·723879	1·381445	·8100416	6
55	·5866080	·724322	1·380600	·8098710	5
56	·5868435	·724766	1·379755	·8097004	4
57	·5870790	·725210	1·378910	·8095296	3
58	·5873145	·725654	1·378067	·8093588	2
59	·5875499	·726098	1·377224	·8091879	1
60	·5877853	·726542	1·376381	·8090170	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 54.

NATURAL SINES AND TANGENTS TO A RADIUS

36 Deg.						36 Deg.						36 Deg.					
′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·5877853	·726542	1·376381	·8090170	60	21	·5927163	·735917	1·358848	·8054113	39	41	·5973919	·744924	1·342417	·8019495	19
1	·5880206	·726987	1·375540	·8088460	59	22	·5929505	·736366	1·358020	·8052389	38	42	·5976251	·745377	1·341602	·8017756	18
2	·5882558	·727431	1·374699	·8086749	58	23	·5931847	·736814	1·357193	·8050664	37	43	·5978583	·745829	1·340788	·8016018	17
3	·5884910	·727876	1·373859	·8085037	57	24	·5934189	·737263	1·356367	·8048938	36	44	·5980915	·746282	1·339975	·8014278	16
4	·5887262	·728321	1·373019	·8083325	56	25	·5936530	·737712	1·355541	·8047211	35	45	·5983246	·746735	1·339162	·8012538	15
5	5889613	·728767	1·372180	·8081612	55	26	·5938871	·738162	1·354716	·8045484	34	46	·5985577	·747188	1·338350	·8010797	14
6	·5891964	·729212	1·371342	·8079899	54	27	·5941211	·738611	1·353891	·8043756	33	47	·5987906	·747642	1·337538	·8009056	13
7	·5894314	·729658	1·370504	·8078185	53	28	·5943550	·739061	1·353068	·8042028	32	48	·5990236	·748095	1·336727	·8007314	12
8	·5896663	·730104	1·369667	·8076470	52	29	·5945889	·739511	1·352244	·8040299	31	49	·5992565	·748549	1·335917	·8005571	11
9	·5899012	·730550	1·368831	·8074754	51	30	·5948228	·739961	1·351422	·8038569	30	50	·5994893	·749003	1·335107	·8003827	10
10	·5901361	·730996	1·367995	·8073038	50	31	·5950566	·740411	1·350600	·8036838	29	51	·5997221	·749457	1·334298	·8002083	9
11	·5903709	·731442	1·367161	·8071321	49	32	·5952904	·740861	1·349779	·8035107	28	52	·5999549	·749911	1·333490	·8000338	8
12	·5906057	·731889	1·366326	·8069603	48	33	·5955241	·741312	1·348958	·8033375	27	53	·6001876	·750366	1·332682	·7998593	7
13	·5908404	·732336	1·365493	·8067885	47	34	·5957577	·741763	1·348139	·8031642	26	54	·6004202	·750821	1·331875	·7996847	6
14	·5910750	·732783	1·364660	·8066166	46	35	·5959913	·742214	1·347319	·8029909	25	55	·6006528	·751276	1·331068	·7995100	5
15	·5913096	·733230	1·363827	·8064446	45	36	·5962249	·742665	1·346501	·8028175	24	56	·6008854	·751731	1·330262	·7993352	4
16	·5915442	·733677	1·362996	·8062726	44	37	·5964584	·743117	1.345683	·8026440	23	57	·6011179	·752186	1·329457	·7991604	3
17	·5917787	·734125	1·362165	·8061005	43	38	·5966918	·743568	1.344865	·8024705	22	58	·6013503	·752642	1·328652	·7989855	2
18	·5920132	·734573	1·361335	·8059283	42	39	·5969252	·744020	1.344049	·8022969	21	59	·6015827	·753098	1·327848	·7988105	1
19	·5922476	·735021	1·360505	·8057560	41	40	·5971586	·744472	1.343233	·8021232	20	60	·6018150	·753554	1·327044	·7986355	0
20	·5924819	·735469	1·359676	·8055837	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′
				Deg. 53.						Deg. 53						Deg. 53.	

NATURAL SINES AND TANGENTS TO A RADIUS 1.

37 Deg. 37 Deg. 37 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·6018150	·753554	1·327044	·7986355	60	21	·6066824	·763175	1·310314	·7949444	39	41	·6112969	·772423	1·294627	·7914014	19
1	·6020473	·754010	1·326242	·7984604	59	22	·6069136	·763636	1·309523	·7947678	38	42	·6115270	·772887	1·293848	·7912235	18
2	·6022795	·754466	1·325439	·7982853	58	23	·6071447	·764096	1·308734	·7945913	37	43	·6117572	·773352	1·293071	·7910456	17
3	.6025117	·754923	1·324638	·7981100	57	24	·6073758	·764557	1·307945	·7944146	36	44	·6119873	·773817	1·292294	·7908676	16
4	·6027439	·755379	1·323837	·7979347	56	25	·6076069	·765018	1·307157	·7942379	35	45	·6122173	·774282	1·291517	·7906896	15
5	·6029760	·755836	1·323036	·7977594	55	26	·6078379	·765480	1·306369	·7940611	34	46	·6124473	·774748	1·290742	·7905115	14
6	·6032080	·756294	1·322237	·7975839	54	27	·6080689	·765941	1·305582	·7938843	33	47	·6126772	·775213	1·289966	·7903333	13
7	·6034400	·756751	1·321437	·7974084	53	28	·6082998	·766403	1·304796	·7937074	32	48	·6129071	·775679	1·289192	·7901550	12
8	·6036719	·757209	1·320639	·7972329	52	29	·6085306	·766864	1·304010	·7935304	31	49	·6131369	·776145	1·288418	·7899767	11
9	·6039038	·757666	1·319841	·7970572	51	30	·6087614	·767327	1·303225	·7933533	30	50	·6133666	·776611	1·287644	·7897983	10
10	·6041356	·758124	1·319044	·7968815	50	31	·6089922	·767789	1·302440	·7931762	29	51	·6135964	·777078	1·286871	·7896198	9
11	·6043674	·758582	1·318247	·7967058	49	32	·6092229	·768251	1·301656	·7929990	28	52	·6138260	·777544	1·286099	·7894413	8
12	·6045991	·759041	1·317451	·7965299	48	33	·6094535	·768714	1·300873	·7928218	27	53	·6140556	·778011	1·285327	·7892627	7
13	·6048308	·759499	1·316655	·7963540	47	34	·6096841	·769177	1·300090	·7926445	26	54	·6142852	·778478	1·284556	·7890841	6
14	·6050624	·759958	1·315861	·7961780	46	35	·6099147	·769640	1·299308	·7924671	25	55	·6145147	·778946	1·283786	·7889054	5
15	·6052940	·760417	1.315066	·7960020	45	36	·6101452	·770103	1·298526	·7922896	24	56	·6147442	·779413	1·283016	·7887266	4
16	·6055255	·760876	1·314273	·7958259	44	37	·6103756	·770567	1·297745	·7921121	23	57	·6149736	·779881	1·282246	·7885477	3
17	·6057570	·761336	1.313480	·7956497	43	38	·6106060	·771030	1·296964	·7919345	22	58	·6152029	·780349	1·281477	·7883688	2
18	·6059884	·761795	1.312687	·7954735	42	39	·6108363	·771494	1·296185	·7917569	21	59	·6154322	·780817	1·280709	·7881898	1
19	·6062198	·762255	1.311895	.7952972	41	40	·6110666	·771958	1·295405	·7915792	20	60	·6156615	·781285	1·279941	·7880108	0
20	·6064511	·762715	1.311104	·7951208	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 52· Deg. 52. Deg. 52.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

40 Deg.						40 Deg.						40 Deg.					
′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·6427876	·839099	1·191753	·7660444	60	21	·6474551	·849563	1·177075	·7621036	39	41	·6518778	·859629	1·163291	·7583240	19
1	·6430104	·839595	1·191049	·7658574	59	22	·6476767	·850064	1·176382	·7619152	38	42	·6520984	·860135	1·162607	·7581343	18
2	·6432332	·840091	1.190346	·7656704	58	23	·6478984	·850565	1·175688	·7617268	37	43	6523189	·860641	1·161923	·7579446	17
3	·6434559	·840587	1·189643	·7654832	57	24	·6481199	·851066	1·174996	·7615383	36	44	·6525394	·861148	1·161240	·7577548	16
4	·6436785	·841084	1·188941	·7652960	56	25	·6483414	·851568	1·174303	·7613497	35	45	·6527598	·861655	1·160557	·7575650	15
5	·6439011	·841581	1·188239	·7651087	55	26	·6485628	·852070	1·173612	·7611611	34	46	·6529801	·862162	1·159874	·7573751	14
6	·6441236	·842078	1·187538	·7649214	54	27	·6487842	·852572	1·172920	·7609724	33	47	·6532004	862669	1·159192	7571851	13
7	·6443461	·842575	1·186837	·7647340	53	28	·6490056	·853075	1·172229	·7607837	32	48	·6534206	·863176	1·158511	·7569951	12
8	·6445685	·843073	1·186136	·7645465	52	29	·6492268	·853577	1·171539	·7605949	31	49	·6536408	·863684	1·157830	·7568050	11
9	·6447909	·843570	1·185437	·7643590	51	30	·6494480	·854080	1·170849	·7604060	30	50	·6538609	·864192	1·157149	·7566148	10
10	·6450132	·844068	1·184737	·7641714	50	31	·6496692	·854583	1·170160	·7602170	29	51	·6540810	·864700	1·156469	·7564246	9
11	·6452355	·844567	1·184038	·7639838	49	32	·6498903	·855087	1·169471	·7600280	28	52	·6543010	·865209	1·155789	·7562343	8
12	·6454577	·845065	1·183340	·7637960	48	33	·6501114	·855591	1·168782	·7598389	27	53	·6545209	·865718	1·155110	·7560439	7
13	·6456798	·845564	1·182642	·7636082	47	34	·6503324	·856095	1·168094	·7596498	26	54	·6547408	·866227	1·154431	·7558535	6
14	·6459019	·846063	1·181944	·7634204	46	35	·6505533	·856599	1·167407	·7594606	25	55	·6549607	·866736	1·153753	·7556630	5
15	·6461240	·846562	1·181247	·7632325	45	36	·6507742	·857103	1·166720	·7592713	24	56	·6551804	·867246	1·153075	·7554724	4
16	·6463460	·847062	1·180551	·7630445	44	37	·6509951	·857608	1·166033	·7590820	23	57	·6554002	·867755	1·152397	·7552818	3
17	·6465679	·847561	1·179855	·7628564	43	38	·6512158	·858113	1·165347	·7588926	22	58	·6556198	·868265	1·151721	·7550911	2
18	·6467898	·848061	1·179159	·7626683	42	39	·6514366	·858618	1·164661	·7587031	21	59	·6558395	·868776	1·151044	·7549004	1
19	·6470116	·848561	1·178464	·7624802	41	40	·6516572	·859124	1·163976	·7585136	20	60	·6560590	·869286	1·150368	·7547096	0
20	·6472334	·849062	1·177769	·7622919	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′
				Deg. 49.						Deg. 49						Deg. 49.	

NATURAL SINES AND TANGENTS TO A RADIUS 1.

41 Deg. 41 Deg. 41 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·6560590	·869286	1·150368	·7547096	60	21	·6606570	·880068	1·136274	·7506879	39	41	·6650131	·890445	1·123032	·7468317	19
1	·6562785	·869797	1·149692	·7545187	59	22	·6608754	·880585	1·135608	·7504957	38	42	·6652304	·890967	1·122375	·7466382	18
2	·6564980	·870308	1·149017	·7543278	58	23	·6610936	·881101	1·134942	·7503034	37	43	·6654475	·891489	1·121718	·7464446	17
3	·6567174	·870820	1·148342	·7541368	57	24	·6613119	·881618	1·134277	·7501111	36	44	·6656646	·892011	1·121061	·7462510	16
4	·6569367	·871331	1·147668	·7539457	56	25	·6615300	·882135	1·133612	·7499187	35	45	·6658817	·892534	1·120405	·7460574	15
5	·6571560	·871843	1·146994	·7537546	55	26	·6617482	·882653	1·132947	·7497262	34	46	·6660987	·893056	1·119749	·7458636	14
6	·6573752	·872355	1·146321	·7535634	54	27	·6619662	·883170	1·132283	·7495337	33	47	·6663156	·893579	1·119094	·7456699	13
7	·6575944	·872868	1·145648	·7533721	53	28	·6621842	·883688	1·131620	·7493411	32	48	·6665325	·894103	1·118439	·7454760	12
8	·6578135	·873380	1·144976	·7531808	52	29	·6624022	·884206	1·130957	·7491484	31	49	·6667493	.894626	1·117784	·7452821	11
9	·6580326	·873893	1·144304	·7529894	51	30	·6626200	·884725	1·130294	·7489557	30	50	·6669661	·895150	1·117130	·7450881	10
10	·6582516	·874406	1·143632	·7527980	50	31	·6628379	·885244	1·129632	·7487629	29	51	·6671828	·895674	1·116476	·7448941	9
11	·6584706	·874920	1·142961	·7526065	49	32	·6630557	·885763	1·128970	·7485701	28	52	·6673994	·896199	1·115823	·7446999	8
12	·6586895	·875433	1·142290	·7524149	48	33	·6632734	·886282	1·128308	·7483772	27	53	·6676160	·896723	1·115170	·7445058	7
13	·6589083	·875947	1·141620	·7522233	47	34	·6634910	·886801	1·127647	·7481842	26	54	·6678326	·897248	1·114518	·7443115	6
14	·6591271	·876462	1·140950	·7520316	46	35	·6637087	·887321	1·126987	·7479912	25	55	·6680490	·897773	1·113866	·7441173	5
15	·6593458	·876976	1·140281	·7518398	45	36	·6639262	·887841	1·126327	·7477981	24	56	·6682655	·898299	1·113214	·7439229	4
16	·6595645	·877491	1·139612	·7516480	44	37	·6641437	·888361	1·125667	·7476049	23	57	·6684818	·898825	1·112563	·7437285	3
17	·6597831	·878006	1·138944	·7514561	43	38	·6643612	·888882	1·125008	·7474117	22	58	·6686981	·899351	1·111912	·7435340	2
18	·6600017	·878521	1·138276	·7512641	42	39	·6645785	·889403	1·124349	·7472184	21	59	·6689144	·899877	1·111262	·7433394	1
19	·6602202	·879037	1·137608	·7510721	41	40	·6647959	·889924	1·123690	·7470251	20	60	·6691306	·900404	1·110612	·7431448	0
20	·6604386	·879552	1·136941	·7508800	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 48. Deg. 48. Deg. 48.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

42 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·6691306	·900404	1·110612	·7431448	60
1	·6693468	·900930	1·109963	·7429502	59
2	·6695628	·901458	1·109314	·7427554	58
3	·6697789	·901985	1·108665	·7425606	57
4	·6699948	·902513	1·108017	·7423658	56
5	·6702108	·903041	1·107369	·7421708	55
6	·6704266	·903569	1·106721	·7419758	54
7	·6706424	904097	1·106075	·7417808	53
8	·6708582	·904626	1·105428	·7415857	52
9	·6710739	·905155	1·104782	·7413905	51
10	·6712895	·905685	1·104136	·7411953	50
11	·6715051	·906214	1·103491	·7410000	49
12	·6717206	·906744	1·102846	·7408046	48
13	·6719361	·907274	1·102201	·7406092	47
14	·6721515	·907805	1·101557	·7404137	46
15	·6723668	·908336	1·100914	·7402181	45
16	·6725821	·908867	1·100270	·7400225	44
17	·6727973	·909398	1·099628	·7398268	43
18	·6730125	·909930	1·098985	·7396311	42
19	·6732276	·910461	1·098343	·7394353	41
20	·6734427	·910994	1·097702	·7392394	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 47.

42 Deg.

′	Sine.	Tang.	Cotang.		′
21	·6736577	·911526	1·097060	·7390435	39
22	·6738727	·912059	1·096420	·7388475	38
23	·6740876	·912592	1·095779	·7386515	37
24	·6743024	·913125	1·095139	·7384553	36
25	·6745172	·913659	1·094500	·7382592	35
26	·6747319	·914192	1·093861	·7380629	34
27	·6749466	·914727	1·093222	·7378666	33
28	·6751612	·915261	1·092584	·7376703	32
29	·6753757	·915796	1·091946	·7374738	31
30	·6755902	·916331	1·091308	·7372773	30
31	·6758046	·916866	1·090671	·7370808	29
32	·6760190	·917402	1·090034	·7368842	28
33	·6762333	·917937	1·089398	·7366875	27
34	·6764476	·918474	1·088762	·7364908	26
35	·6766618	·919010	1·088126	·7362940	25
36	·6768760	·919547	1·087491	·7360971	24
37	·6770901	·920084	1·086857	·7359002	23
38	·6773041	·920621	1·086222	·7357032	22
39	·6775181	·921159	1·085588	·7355061	21
40	·6777320	·921696	1·084955	·7353090	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 47.

42 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·6779459	·922235	1.084322	·7351118	19
42	·6781597	·922773	1·083689	·7349146	18
43	·6783734	·923312	1·083057	·7347173	17
44	·6785871	·923851	1·082425	·7345199	16
45	·6788007	·924390	1·081793	·7343225	15
46	·6790143	·924930	1·081162	·7341250	14
47	·6792278	·925470	1·080532	·7339275	13
48	·6794413	·926010	1·079901	·7337299	12
49	·6796547	·926550	1·079271	·7335322	11
50	·6798681	·927091	1·078642	·7333345	10
51	·6800813	·927632	1·078013	·7331367	9
52	·6802946	·928173	1·077384	·7329388	8
53	·6805078	·928715	1·076756	·7327409	7
54	·6807209	·929257	1·076128	·7325429	6
55	·6809339	·929799	1·075500	·7323449	5
56	·6811469	·930342	1·074873	·7321467	4
57	·6813599	·930884	1·074246	·7319486	3
58	·6815728	·931428	1·073620	·7317503	2
59	·6817856	·931971	1·072994	·7315521	1
60	·6819984	·932515	1·072368	·7313537	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 47.

NATURAL SINES AND TANGENTS TO A RADIUS 1.

43 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
0	·6819984	·932515	1·072368	·7313537	60
1	·6822111	·933059	1·071743	·7311553	59
2	·6824237	·933603	1·071118	·7309568	58
3	·6826363	·934147	1·070494	·7307583	57
4	·6828489	·934692	1·069870	·7305597	56
5	·6830613	·935238	1·069246	·7303610	55
6	·6832738	·935783	1·068623	·7301623	54
7	·6834861	·936329	1·068000	·7299635	53
8	·6836984	·936875	1·067377	·7297646	52
9	·6839107	·937421	1·066755	·7295657	51
10	·6841229	·937968	1·066134	·7293668	50
11	·6843350	·938515	1·065512	·7291677	49
12	·6845471	·939062	1·064891	·7289686	48
13	·6847591	·939610	1·064271	·7287695	47
14	·6849711	·940157	1·063651	·7285703	46
15	·6851830	·940706	1·063031	·7283710	45
16	·6853948	·941254	1·062411	·7281716	44
17	·6856066	·941803	1·061792	·7279722	43
18	·6858184	·942352	1·061174	·7277728	42
19	·6860300	·942901	1·060556	·7275732	41
20	·6862416	·943451	1·059938	·7273736	40
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 46.

43 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
21	·6864532	·944001	1·059320	·7271740	39
22	·6866647	·944551	1·058703	·7269743	38
23	·6868761	·945102	1·058086	·7267745	37
24	·6870875	·945653	1·057470	·7265747	36
25	·6872988	·946204	1·056854	·7263748	35
26	·6875101	·946755	1·056238	·7261748	34
27	·6877213	·947307	1·055623	·7259748	33
28	·6879325	·947859	1·055008	·7257747	32
29	·6881435	·948411	1·054394	·7255746	31
30	·6883546	·948964	1·053780	·7253744	30
31	·6885655	·949517	1·053166	·7251741	29
32	·6887765	·950070	1·052553	·7249738	28
33	·6889873	·950624	1·051940	·7247734	27
34	·6891981	·951178	1·051327	·7245729	26
35	·6894089	·951732	1·050715	·7243724	25
36	·6896195	·952287	1·050103	·7241719	24
37	·6898302	·952842	1·049492	·7239712	23
38	·6900407	·953397	1·048880	·7237705	22
39	·6902512	·953952	1·048270	·7235698	21
40	·6904617	·954508	1·047659	·7233690	20
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 46.

43 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′
41	·6906721	·955064	1·047049	·7231681	19
42	·6908824	·955620	1·046440	·7229671	18
43	·6910927	·956177	1·045831	·7227661	17
44	·6913029	·956734	1·045222	·7225651	16
45	·6915131	·957291	1·044613	·7223640	15
46	·6917232	·957849	1·044005	·7221628	14
47	·6919332	·958407	1·043397	·7219615	13
48	·6921432	·958965	1·042790	·7217602	12
49	·6923531	·959524	1·042183	·7215589	11
50	·6925630	·960082	1·041576	·7213574	10
51	·6927728	·960642	1·040970	·7211559	9
52	·6929825	·961201	1·040364	·7209544	8
53	·6931922	·961761	1·039758	·7207528	7
54	·6934018	·962321	1·039153	·7205511	6
55	·6936114	·962881	1·038548	·7203494	5
56	·6938209	·963442	1·037944	·7201476	4
57	·6940304	·964003	1·037340	·7199457	3
58	·6942398	·964565	1·036736	·7197438	2
59	·6944491	·965126	1·036133	·7195418	1
60	·6946584	·965688	1·035530	·7193398	0
′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 46

NATURAL SINES AND TANGENTS TO A RADIUS .

44 Deg. 44 Deg. 44 Deg.

′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	·6946584	·965688	1·035530	·7193398	60	21	·6990396	·977564	1·022950	·7150830	39	41	·7031879	·989006	1·011115	·7110041	19
1	·6948676	·966251	1·034927	·7191377	59	22	·6992476	·978133	1·022355	·7148796	38	42	·7033947	·989582	1·010527	·7107995	18
2	·6950767	·966813	1·034325	·7189355	58	23	·6994555	·978702	1·021760	·7146762	37	43	·7036014	·990158	1·009939	·7105948	17
3	6952858	·967376	1·033723	·7187333	57	24	·6996633	·979272	1·021166	·7144727	36	44	·7038081	·990734	1·009352	·7103901	16
4	·6954949	·967939	1·033122	·7185310	56	25	·6998711	·979842	1·020572	·7142691	35	45	·7040147	·991311	1·008764	·7101854	15
5	·6957039	·968503	1·032520	·7183287	55	26	·7000789	·980412	1·019978	·7140655	34	46	·7042213	·991888	1·008178	·7099806	14
6	·6959128	·969067	1·031919	·7181263	54	27	·7002866	·980983	1·019385	·7138618	33	47	·7044278	·992465	1·007591	·7097757	13
7	·6961217	·969631	1·031319	·7179238	53	28	·7004942	·981554	1·018792	·7136581	32	48	·7046342	·993042	1·007005	·7095707	12
8	·6963305	·970196	1·030719	·7177213	52	29	·7007018	·982125	1·018199	·7134543	31	49	·7048406	·993620	1·006420	·7093657	11
9	·6965392	·970761	1·030119	·7175187	51	30	·7009093	·982697	1·017607	·7132504	30	50	·7050469	·994199	1·005834	·7091607	10
10	·6967479	·971326	1·029520	·7173161	50	31	·7011167	·983269	1·017015	·7130465	29	51	·7052532	·994777	1·005249	·7089556	9
11	·6969565	·971891	1·028921	·7171134	49	32	·7013241	·983841	1·016423	·7128426	28	52	·7054594	·995356	1·004665	·7087504	8
12	·6971651	·972457	1·028322	·7169106	48	33	·7015314	·984414	1·015832	·7126385	27	53	·7056655	·995935	1·004080	·7085451	7
13	·6973736	·973023	1·027724	·7167078	47	34	·7017387	·984987	1·015241	·7124344	26	54	·7058716	·996515	1·003496	·7083398	6
14	·6975821	·973590	1·027126	·7165049	46	35	·7019459	·985560	1·014651	·7122303	25	55	·7060776	·997095	1·002913	·7081345	5
15	·6977905	·974156	1·026528	·7163019	45	36	·7021531	·986133	1·014061	·7120260	24	56	·7062835	·997675	1·002329	·7079291	4
16	·6979988	·974724	1·025931	·7160989	44	37	·7023601	.986707	1·013471	·7118218	23	57	·7064894	·998256	1·001746	·7077236	3
17	·6982071	·975291	1·025334	·7158959	43	38	·7025672	.987282	1·012881	·7116174	22	58	·7066953	·998837	1·001164	·7075180	2
18	·6984153	·975859	1·024738	·7156927	42	39	·7027741	.987856	1·012292	·7114130	21	59	·7069011	·999418	1·000581	·7073124	1
19	·6986234	·976427	1·024141	·7154895	41	40	·7029811	.988431	1·011703	·7112086	20	60	·7071068	1·00000	1·000000	·7071068	0
20	·6988315	·976995	1·023546	·7152863	40												
′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′	′	Cosine.	Cotang.	Tang.	Sine.	′

Deg. 45. Deg. 45. Deg. 45.

ARTICLE XLVI.

Versed Sines.

The **Versed Sine** of an angle or arc, A B, Fig. 58, is that part, W A, of the diameter, D A, which is between the sine, B W, and the extremity, A, of the arc.

Thus,

W A (=D T) is the versed sine of $\overset{60^\circ}{AB}$ and of $\overset{300^\circ}{BCDEFA}$.

W D (=T A) is the versed sine of $\overset{120^\circ}{BCD}$ and of $\overset{240^\circ}{ABCDE}$.

The versed sine is=radius *minus* cosine. (See p. 121.) But in the second quadrant C D (counting from A), or in angles of between 90° and 180°, and in the third quadrant D F, or between 180° and 270°, the cosines T X etc., extend from the center to the *left*, and are regarded as *negative* or *minus*. Hence in angles of more than 90° and less than 270°, the *numerical* value of the versed sine is radius *plus* cosine.

On pages 170 to 192 is a **Table of Natural Versed Sines** (those of circles whose radius is 1) for all angles from 0° to 360°.

The versed sines of angles increase from 0 at 0° to 2 at 180°; and then decrease from 2 at 180° to 0 at 360°. The angles are read *downward* on the *left* of the column, from 0° to 180°; and *upward* on the *right*, from 180° to 360°. Each versed sine thus corresponds to *two* angles, and when an *angle* is to be found from the table by means of its versed sine, we must decide from the circumstances of the case which of the two is the angle required. See Remark, p. 123.

To find the versed sine of an angle containing an odd number of minutes, take the mean between those immediately above and below it in the table. Thus, to find the versed sine of 89° 57′ (the versed sines vary most rapidly at 90° and at 270°), we find in the table the versed sine of 89° 56′ (·9988) and that of 89° 58′ (·9994); and the mean between these two, or ·9991, is the required versed sine of 89° 57′.

° ′		° ′	° ′		° ′	° ′		° ′	° ′		° ′
16 0	·0387	60	**18** 0	·0489	60	**20** 0	·0603	60	**22** 0	·0728	60
2	·0389	58	2	·0491	58	2	·0605	58	2	·0730	58
4	·0391	56	4	·0493	56	4	·0607	56	4	·0733	56
6	·0392	54	6	·0495	54	6	·0609	54	6	·0735	54
8	·0394	52	8	·0497	52	8	·0611	52	8	·0737	52
10	·0395	50	10	·0498	50	10	·0613	50	10	·0739	50
12	·0397	48	12	·0500	48	12	·0615	48	12	·0741	48
14	·0399	46	14	·0502	46	14	·0617	46	14	·0743	46
16	·0400	44	16	·0504	44	16	·0619	44	16	·0746	44
18	·0402	42	18	·0506	42	18	·0621	42	18	·0748	42
20	·0404	40	20	·0508	40	20	·0623	40	20	·0750	40
22	·0405	38	22	·0509	38	22	·0625	38	22	·0752	38
24	·0407	36	24	·0511	36	24	·0627	36	24	·0755	36
26	·0409	34	26	·0513	34	26	·0629	34	26	·0757	34
28	·0410	32	28	·0515	32	28	·0631	32	28	·0759	32
30	·0412	30	30	·0517	30	30	·0633	30	30	·0761	30
32	·0413	28	32	·0519	28	32	·0635	28	32	·0763	28
34	·0415	26	34	·0520	26	34	·0637	26	34	·0766	26
36	·0417	24	36	·0522	24	36	·0639	24	36	·0768	24
38	·0418	22	38	·0524	22	38	0641	22	38	·0770	22
40	·0420	20	40	·0526	20	40	·0644	20	40	·0772	20
42	·0422	18	42	·0528	18	42	·0646	18	42	·0775	18
44	·0423	16	44	·0530	16	44	·0648	16	44	·0777	16
46	·0425	14	46	·0532	14	46	·0650	14	46	·0779	14
48	·0427	12	48	·0534	12	48	·0652	12	48	·0781	12
50	·0428	10	50	·0535	10	50	·0654	10	50	·0784	10
52	·0430	8	52	·0537	8	52	·0656	8	52	·0786	8
54	·0432	6	54	·0539	6	54	·0658	6	54	·0788	6
56	·0434	4	56	·0541	4	56	·0660	4	56	·0790	4
58	·0435	2	58	·0543	2	58	·0662	2	58	·0793	2
60	·0437	**343** 0	60	·0545	**341** 0	60	·0664	**339** 0	60	·0795	**337** 0

° ′		° ′	° ′		° ′	° ′		° ′	° ′		° ′
17 0	·0437	60	**19** 0	·0545	60	**21** 0	·0664	60	**23** 0	·0795	60
2	·0439	58	2	·0547	58	2	·0666	58	2	·0797	58
4	·0440	56	4	·0549	56	4	·0668	56	4	·0800	56
6	·0442	54	6	·0551	54	6	·0670	54	6	·0802	54
8	·0444	52	8	·0552	52	8	·0673	52	8	·0804	52
10	·0445	50	10	·0554	50	10	·0675	50	10	·0806	50
12	·0447	48	12	·0556	48	12	·0677	48	12	·0809	48
14	·0449	46	14	·0558	46	14	·0679	46	14	·0811	46
16	·0451	44	16	·0560	44	16	·0681	44	16	·0813	44
18	·0452	42	18	·0562	42	18	·0683	42	18	·0816	42
20	·0454	40	20	·0564	40	20	·0685	40	20	·0818	40
22	·0456	38	22	·0566	38	22	·0687	38	22	·0820	38
24	·0458	36	24	·0568	36	24	·0689	36	24	·0822	36
26	·0459	34	26	·0570	34	26	·0692	34	26	·0825	34
28	·0461	32	28	·0572	32	28	·0694	32	28	·0827	32
30	·0463	30	30	·0574	30	30	·0696	30	30	·0829	30
32	·0465	28	32	·0576	28	32	·0698	28	32	·0832	28
34	·0466	26	34	·0577	26	34	·0700	26	34	·0834	26
36	·0468	24	36	·0579	24	36	·0702	24	36	·0836	24
38	·0470	22	38	·0581	22	38	·0704	22	38	·0839	22
40	·0472	20	40	·0583	20	40	·0707	20	40	·0841	20
42	·0473	18	42	·0585	18	42	·0709	18	42	·0843	18
44	·0475	16	44	·0587	16	44	·0711	16	44	0846	16
46	·0477	14	46	·0589	14	46	·0713	14	46	·0848	14
48	·0479	12	48	·0591	12	48	·0715	12	48	·0850	12
50	·0480	10	50	·0593	10	50	·0717	10	50	·0853	10
52	·0482	8	52	·0595	8	52	·0719	8	52	·0855	8
54	·0484	6	54	·0597	6	54	·0722	6	54	·0857	6
56	·0486	4	56	·0599	4	56	·0724	4	56	·0860	4
58	·0488	2	58	·0601	2	58	·0726	2	58	·0862	2
60	·0489	**342** 0	60	·0603	**340** 0	60	·0728	**338** 0	60	·0865	**336** 0

° ′		° ′	° ′		° ′	° ′		° ′	° ′		° ′
24 0	·0865	60	**26** 0	·1012	60	**28** 0	·1171	60	**30** 0	·1340	60
2	·0867	58	2	·1015	58	2	·1173	58	2	·1343	58
4	·0869	56	4	·1017	56	4	·1176	56	4	·1346	56
6	·0872	54	6	·1020	54	6	·1179	54	6	·1348	54
8	·0874	52	8	·1022	52	8	·1181	52	8	·1351	52
10	·0876	50	10	·1025	50	10	·1184	50	10	·1354	50
12	·0879	48	12	·1027	48	12	·1187	48	12	·1357	48
14	·0881	46	14	·1030	46	14	·1190	46	14	·1360	46
16	·0884	44	16	·1033	44	16	·1192	44	16	·1363	44
18	·0886	42	18	·1035	42	18	·1195	42	18	·1366	42
20	·0888	40	20	·1038	40	20	·1198	40	20	·1369	40
22	·0891	38	22	·1040	38	22	·1201	38	22	·1372	38
24	·0893	36	24	·1043	36	24	·1204	36	24	·1375	36
26	·0896	34	26	·1045	34	26	·1206	34	26	·1378	34
28	·0898	32	28	·1048	32	28	·1209	32	28	·1381	32
30	·0900	30	30	·1051	30	30	·1212	30	30	·1384	30
32	·0903	28	32	·1053	28	32	·1215	28	32	·1387	28
34	·0905	26	34	·1056	26	34	·1217	26	34	·1390	26
36	·0908	24	36	·1058	24	36	·1220	24	36	·1393	24
38	·0910	22	38	·1061	22	38	·1223	22	38	·1396	22
40	·0912	20	40	·1064	20	40	·1226	20	40	·1399	20
42	·0915	18	42	·1066	18	42	·1229	18	42	·1401	18
44	·0917	16	44	·1069	16	44	·1231	16	44	·1404	16
46	·0920	14	46	·1072	14	46	·1234	14	46	·1407	14
48	·0922	12	48	·1074	12	48	·1237	12	48	·1410	12
50	·0925	10	50	·1077	10	50	·1240	10	50	·1413	10
52	·0927	8	52	·1079	8	52	·1243	8	52	·1416	8
54	·0930	6	54	·1082	6	54	·1245	6	54	·1419	6
56	·0932	4	56	·1085	4	56	·1248	4	56	·1422	4
58	·0934	2	58	·1087	2	58	·1251	2	58	·1425	2
60	·0937	**335** 0	60	·1090	**333** 0	60	·1254	**331** 0	60	·1428	**329** 0
° ′		° ′	° ′		° ′	° ′		° ′	° ′		° ′
25 0	·0937	60	**27** 0	·1090	60	**29** 0	·1254	60	**31** 0	·1428	60
2	·0939	58	2	·1093	58	2	·1257	58	2	·1431	58
4	·0942	56	4	·1095	56	4	·1259	56	4	·1434	56
6	·0944	54	6	·1098	54	6	·1262	54	6	·1437	54
8	·0947	52	8	·1101	52	8	·1265	52	8	·1440	52
10	·0949	50	10	·1103	50	10	·1268	50	10	·1443	50
12	·0952	48	12	·1106	48	12	·1271	48	12	·1446	48
14	·0954	46	14	·1108	46	14	·1274	46	14	·1449	46
16	·0957	44	16	·1111	44	16	·1276	44	16	·1452	44
18	·0959	42	18	·1114	42	18	·1279	42	18	·1455	42
20	·0962	40	20	·1116	40	20	·1282	40	20	·1458	40
22	·0964	38	22	·1119	38	22	·1285	38	22	·1461	38
24	·0967	36	24	·1122	36	24	·1288	36	24	·1464	36
26	·0969	34	26	·1125	34	26	·1291	34	26	·1468	34
28	·0972	32	28	·1127	32	28	·1294	32	28	·1471	32
30	·0974	30	30	·1130	30	30	·1296	30	30	·1474	30
32	·0977	28	32	·1133	28	32	·1299	28	32	·1477	28
34	·0979	26	34	·1135	26	34	·1302	26	34	·1480	26
36	·0982	24	36	·1138	24	36	·1305	24	36	·1483	24
38	·0984	22	38	·1141	22	38	·1308	22	38	·1486	22
40	·0987	20	40	·1143	20	40	·1311	20	40	·1489	20
42	·0989	18	42	·1146	18	42	·1314	18	42	·1492	18
44	·0992	16	44	·1149	16	44	·1317	16	44	·1495	16
46	·0994	14	46	·1151	14	46	·1319	14	46	·1498	14
48	·0997	12	48	·1154	12	48	·1322	12	48	·1501	12
50	·0999	10	50	·1157	10	50	·1325	10	50	·1504	10
52	·1002	8	52	·1160	8	52	·1328	8	52	·1507	8
54	·1004	6	54	·1162	6	54	·1331	6	54	·1510	6
56	·1007	4	56	·1165	4	56	·1334	4	56	·1513	4
58	·1010	2	58	·1168	2	58	·1337	2	58	·1516	2
60	·1012	**334** 0	60	·1171	**332** 0	60	·1340	**330** 0	60	·1520	**328** 0

TABLE OF VERSED SINES.

° ′		° ′	° ′		° ′	° ′		° ′	° ′		° ′
32 0	·1520	60	**34** 0	·1710	60	**36** 0	·1910	60	**38** 0	·2120	60
2	·1523	58	2	·1713	58	2	·1913	58	2	·2123	58
4	·1526	56	4	·1716	56	4	·1917	56	4	·2127	56
6	·1529	54	6	·1719	54	6	·1920	54	6	·2131	54
8	·1532	52	8	·1723	52	8	·1924	52	8	·2134	52
10	·1535	50	10	·1726	50	10	·1927	50	10	·2138	50
12	·1538	48	12	·1729	48	12	·1930	48	12	·2141	48
14	·1541	46	14	·1732	46	14	·1934	46	14	·2145	46
16	·1544	44	16	·1736	44	16	·1937	44	16	·2149	44
18	·1547	42	18	·1739	42	18	·1941	42	18	·2152	42
20	·1550	40	20	·1742	40	20	·1944	40	20	·2156	40
22	·1554	38	22	·1746	38	22	·1948	38	22	·2159	38
24	·1557	36	24	·1749	36	24	·1951	36	24	·2163	36
26	·1560	34	26	·1752	34	26	·1955	34	26	·2167	34
28	·1563	32	28	·1755	32	28	·1958	32	28	·2170	32
30	·1566	30	30	·1759	30	30	·1961	30	30	·2174	30
32	·1569	28	32	·1762	28	32	·1965	28	32	·2178	28
34	·1572	26	34	·1765	26	34	·1968	26	34	·2181	26
36	·1575	24	36	·1769	24	36	·1972	24	36	·2185	24
38	·1579	22	38	·1772	22	38	·1975	22	38	·2188	22
40	·1582	20	40	·1775	20	40	·1979	20	40	·2192	20
42	·1585	18	42	·1779	18	42	·1982	18	42	·2196	18
44	·1588	16	44	·1782	16	44	·1986	16	44	·2199	16
46	·1591	14	46	·1785	14	46	·1989	14	46	·2203	14
48	·1594	12	48	·1789	12	48	·1993	12	48	·2207	12
50	·1597	10	50	·1792	10	50	·1996	10	50	·2210	10
52	·1601	8	52	·1795	8	52	·2000	8	52	·2214	8
54	·1604	6	54	·1798	6	54	·2003	6	54	·2218	6
56	·1607	4	56	·1802	4	56	·2007	4	56	·2221	4
58	·1610	2	58	·1805	2	58	·2010	2	58	·2225	2
60	·1613	**327** 0	60	·1808	**325** 0	60	·2014	**323** 0	60	·2229	**321** 0

° ′		° ′	° ′		° ′	° ′		° ′	° ′		° ′
33 0	·1613	60	**35** 0	·1808	60	**37** 0	·2014	60	**39** 0	·2229	60
2	·1616	58	2	·1812	58	2	·2017	58	2	·2232	58
4	·1620	56	4	·1815	56	4	·2021	56	4	·2236	56
6	·1623	54	6	·1819	54	6	·2024	54	6	·2240	54
8	·1626	52	8	·1822	52	8	·2028	52	8	·2243	52
10	·1629	50	10	·1825	50	10	·2031	50	10	·2247	50
12	·1632	48	12	·1829	48	12	·2035	48	12	·2251	48
14	1636	46	14	·1832	46	14	·2038	46	14	·2254	46
16	·1639	44	16	·1835	44	16	·2042	44	16	·2258	44
18	·1642	42	18	·1839	42	18	·2045	42	18	·2262	42
20	·1645	40	20	·1842	40	20	·2049	40	20	·2265	40
22	·1648	38	22	·1845	38	22	·2052	38	22	·2269	38
24	·1652	36	24	·1849	36	24	·2056	36	24	·2273	36
26	·1655	34	26	·1852	34	26	·2059	34	26	·2276	34
28	·1658	32	28	·1855	32	28	·2063	32	28	·2280	32
30	·1661	30	30	·1859	30	30	·2066	30	30	·2284	30
32	·1664	28	32	·1862	28	32	·2070	28	32	·2287	28
34	·1668	26	34	·1866	26	34	·2074	26	34	·2291	26
36	·1671	24	36	·1869	24	36	·2077	24	36	·2295	24
38	·1674	22	38	·1872	22	38	·2081	22	38	·2299	22
40	·1677	20	40	·1876	20	40	·2084	20	40	·2302	20
42	·1680	18	42	·1879	18	42	·2088	18	42	·2306	18
44	·1684	16	44	·1883	16	44	·2091	16	44	·2310	16
46	·1687	14	46	·1886	14	46	·2095	14	46	·2313	14
48	·1690	12	48	·1889	12	48	·2098	12	48	·2317	12
50	·1693	10	50	·1893	10	50	·2102	10	50	·2321	10
52	·1697	8	52	·1896	8	52	·2106	8	52	·2325	8
54	·1700	6	54	·1900	6	54	·2109	6	54	·2328	6
56	·1703	4	56	·1903	4	56	·2113	4	56	·2332	4
58	·1706	2	58	·1906	2	58	·2116	2	58	·2336	2
60	·1710	**326** 0	60	·1910	**324** 0	60	·2120	**322** 0	60	·2340	**320** 0

° ′		° ′	° ′		° ′	° ′		° ′	° ′		° ′
40 0	·2340	60	**42** 0	·2569	60	**44** 0	·2807	60	**46** 0	·3053	60
2	·2343	58	2	·2572	58	2	·2811	58	2	·3058	58
4	·2347	56	4	·2576	56	4	·2815	56	4	·3062	56
6	·2351	54	6	·2580	54	6	·2819	54	6	·3066	54
8	·2355	52	8	·2584	52	8	·2823	52	8	·3070	52
10	·2358	50	10	·2588	50	10	·2827	50	10	·3074	50
12	·2362	47	12	·2592	48	12	·2831	48	12	·3079	48
14	·2366	46	14	·2596	46	14	·2835	46	14	·3083	46
16	·2370	44	16	·2600	44	16	·2839	44	16	·3087	44
18	·2373	42	18	·2604	42	18	·2843	42	18	·3091	42
20	·2377	40	20	·2608	40	20	·2847	40	20	·3095	40
22	·2381	38	22	·2612	38	22	·2851	38	22	·3100	38
24	·2385	36	24	·2615	36	24	·2855	36	24	·3104	36
26	·2388	34	26	·2619	34	26	·2859	34	26	·3108	34
28	·2392	32	28	·2623	32	28	·2863	32	28	·3112	32
30	·2396	30	30	·2627	30	30	·2867	30	30	·3116	30
32	·2400	28	32	·2631	28	32	·2872	28	32	·3121	28
34	·2404	26	34	·2635	26	34	·2876	26	34	·3125	26
36	·2407	24	36	·2639	24	36	·2880	24	36	·3129	24
38	·2411	22	38	·2643	22	38	·2884	22	38	·3133	22
40	·2415	20	40	·2647	20	40	·2888	20	40	·3138	20
42	·2419	18	42	·2651	18	42	·2892	18	42	·3142	18
44	·2422	16	44	·2655	16	44	·2896	16	44	·3146	16
46	·2426	14	46	·2659	14	46	·2900	14	46	·3150	14
48	·2430	12	48	·2663	12	48	·2904	12	48	·3155	12
50	·2434	10	50	·2667	10	50	·2908	10	50	·3159	10
52	·2438	8	52	·2671	8	52	·2912	8	52	·3163	8
54	·2441	6	54	·2675	6	54	·2917	6	54	·3167	6
56	·2445	4	56	·2679	4	56	·2921	4	56	·3172	4
58	·2449	2	58	·2682	2	58	·2925	2	58	·3176	2
60	·2453	**319** 0	60	·2686	**317** 0	60	·2929	**315** 0	60	·3180	**313** 0

° ′		° ′	° ′		° ′	° ′		° ′	° ′		° ′
41 0	·2453	60	**43** 0	·2686	60	**45** 0	·2929	60	**47** 0	·3180	60
2	·2457	58	2	·2690	58	2	·2933	58	2	·3184	58
4	·2461	56	4	·2694	56	4	·2937	56	4	·3189	56
6	·2464	54	6	·2698	54	6	·2941	54	6	·3193	54
8	·2468	52	8	·2702	52	8	·2945	52	8	·3197	52
10	·2472	50	10	·2706	50	10	·2950	50	10	·3201	50
12	·2476	48	12	·2710	48	12	·2954	48	12	·3206	48
14	·2480	46	14	·2714	46	14	·2958	46	14	·3210	46
16	·2484	44	16	·2718	44	16	·2962	44	16	·3214	44
18	·2487	42	18	·2722	42	18	·2966	42	18	·3218	42
20	·2491	40	20	·2726	40	20	·2970	40	20	·3223	40
22	·2495	38	22	·2730	38	22	·2974	38	22	·3227	38
24	·2499	36	24	·2734	36	24	·2978	36	24	·3231	36
26	·2503	34	26	·2738	34	26	·2983	34	26	·3236	34
28	·2507	32	28	·2742	32	28	·2987	32	28	·3240	32
30	·2510	30	30	·2746	30	30	·2991	30	30	·3244	30
32	·2514	28	32	·2750	28	32	·2995	28	32	·3248	28
34	·2518	26	34	·2754	26	34	·2999	26	34	·3253	26
36	·2522	24	36	·2758	24	36	·3003	24	36	·3257	24
38	·2526	22	38	·2762	22	38	·3008	22	38	·3261	22
40	·2530	20	40	·2766	20	40	·3012	20	40	·3266	20
42	·2534	18	42	·2770	18	42	·3016	18	42	·3270	18
44	·2537	16	44	·2774	16	44	·3020	16	44	·3274	16
46	·2541	14	46	·2778	14	46	·3024	14	46	·3278	14
48	·2545	12	48	·2782	12	48	·3028	12	48	·3283	12
50	·2549	10	50	·2786	10	50	·3033	10	50	·3287	10
52	·2553	8	52	·2790	8	52	·3037	8	52	·3291	8
54	·2557	6	54	·2794	6	54	·3041	6	54	·3296	6
56	·2561	4	56	·2799	4	56	·3045	4	56	·3300	4
58	·2565	2	58	·2803	2	58	·3049	2	58	·3304	2
60	·2569	**318** 0	60	·2807	**316** 0	60	·3053	**314** 0	60	·3309	**312** 0

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48 0	·3309	60	**50** 0	·3572	60	**52** 0	·3843	60	**54** 0	·4122	60
2	·3313	58	2	·3577	58	2	·3848	58	2	·4127	58
4	·3317	56	4	·3581	56	4	·3853	56	4	·4132	56
6	·3322	54	6	·3586	54	6	·3857	54	6	·4136	54
8	·3326	52	8	·3590	52	8	·3862	52	8	·4141	52
10	·3330	50	10	·3594	50	10	·3866	50	10	·4146	50
12	·3335	48	12	·3599	48	12	·3871	48	12	·4150	48
14	·3339	46	14	·3603	46	14	·3876	46	14	·4155	46
16	·3343	44	16	·3608	44	16	·3880	44	16	·4160	44
18	·3348	42	18	·3612	42	18	·3885	42	18	·4165	42
20	·3352	40	20	·3617	40	20	·3889	40	20	·4169	40
22	·3356	38	22	·3621	38	22	·3894	38	22	·4174	38
24	·3361	36	24	·3626	36	24	·3899	36	24	·4179	36
26	·3365	34	26	·3630	34	26	·3903	34	26	·4184	34
28	·3369	32	28	·3635	32	28	·3908	32	28	·4188	32
30	·3374	30	30	·3639	30	30	·3912	30	30	·4193	30
32	·3378	28	32	·3644	28	32	·3917	28	32	·4198	28
34	·3383	26	34	·3648	26	34	·3922	26	34	·4202	26
36	·3387	24	36	·3653	24	36	·3926	24	36	·4207	24
38	·3391	22	38	·3657	22	38	·3931	22	38	·4212	22
40	·3396	20	40	·3662	20	40	·3935	20	40	·4217	20
42	·3400	18	42	·3666	18	42	·3940	18	42	·4221	18
44	·3404	16	44	·3671	16	44	·3945	16	44	·4226	16
46	·3409	14	46	·3675	14	46	·3949	14	46	·4231	14
48	·3413	12	48	·3680	12	48	·3954	12	48	·4236	12
50	·3417	10	50	·3684	10	50	·3959	10	50	·4240	10
52	·3422	8	52	·3689	8	52	·3963	8	52	·4245	8
54	·3426	6	54	·3693	6	54	·3968	6	54	·4250	6
56	·3431	4	56	·3698	4	56	·3973	4	56	·4255	4
58	·3435	2	58	·3702	2	58	·3978	2	58	·4259	2
60	·3439	**311** 0	60	·3707	**309** 0	60	·3982	**307** 0	60	·4264	**305** 0

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49 0	·3439	60	**51** 0	·3707	60	**53** 0	·3982	60	**55** 0	·4264	60
2	·3444	58	2	·3711	58	2	·3986	58	2	·4269	58
4	·3448	56	4	·3716	56	4	·3991	56	4	·4274	56
6	·3453	54	6	·3720	54	6	·3996	54	6	·4279	54
8	·3457	52	8	·3725	52	8	·4000	52	8	·4283	52
10	·3461	50	10	·3729	50	10	·4005	50	10	·4288	50
12	·3466	48	12	·3734	48	12	·4010	48	12	·4293	48
14	·3470	46	14	·3738	46	14	·4014	46	14	·4298	46
16	·3475	44	16	·3743	44	16	·4019	44	16	·4302	44
18	·3479	42	18	·3748	42	18	·4024	42	18	·4307	42
20	·3483	40	20	·3752	40	20	·4028	40	20	·4312	40
22	·3488	38	22	·3757	38	22	·4033	38	22	·4317	38
24	·3492	36	24	·3761	36	24	·4038	36	24	·4322	36
26	·3497	34	26	·3766	34	26	·4042	34	26	·4326	34
28	·3501	32	28	·3770	32	28	·4047	32	28	·4331	32
30	·3506	30	30	·3775	30	30	·4052	30	30	·4336	30
32	·3510	28	32	·3779	28	32	·4056	28	32	·4341	28
34	·3514	26	34	·3784	26	34	·4061	26	34	·4346	26
36	·3519	24	36	·3789	24	36	·4066	24	36	·4350	24
38	·3523	22	38	·3793	22	38	·4070	22	38	·4355	22
40	·3528	20	40	·3798	20	40	·4075	20	40	·4360	20
42	·3532	18	42	·3802	18	42	·4080	18	42	·4365	18
44	·3537	16	44	·3807	16	44	·4085	16	44	·4370	16
46	·3541	14	46	·3811	14	46	·4089	14	46	·4374	14
48	·3545	12	48	·3816	12	48	·4094	12	48	·4379	12
50	·3550	10	50	·3820	10	50	·4099	10	50	·4384	10
52	·3554	8	52	·3825	8	52	·4103	8	52	·4389	8
54	·3559	6	54	·3830	6	54	·4108	6	54	·4394	6
56	·3563	4	56	·3834	4	56	·4113	4	56	·4398	4
58	·3568	2	58	·3839	2	58	·4117	2	58	·4403	2
60	·3572	**310** 0	60	·3843	**308** 0	60	·4122	**306** 0	60	·4408	**304** 0

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56 0	·4408	60	**58** 0	·4701	60	**60** 0	·5000	60	**62** 0	·5305	60
2	·4413	58	2	·4706	58	2	·5005	58	2	·5310	58
4	·4418	56	4	·4711	56	4	·5010	56	4	·5316	56
6	·4423	54	6	·4716	54	6	·5015	54	6	·5321	54
8	·4427	52	8	·4721	52	8	·5020	52	8	·5326	52
10	·4432	50	10	·4725	50	10	·5025	50	10	·5331	50
12	·4437	48	12	4730	48	12	·5030	48	12	·5336	48
14	·4442	46	14	·4735	46	14	·5035	46	14	·5341	46
16	·4447	44	16	·4740	44	16	·5040	44	16	·5346	44
18	·4452	42	18	·4745	42	18	·5045	42	18	·5352	42
20	·4456	40	20	·4750	40	20	·5050	40	20	·5357	40
22	·4461	38	22	·4755	38	22	·5056	38	22	·5362	38
24	·4466	36	24	·4760	36	24	·5061	36	24	·5367	36
26	·4471	34	26	·4765	34	26	·5066	34	26	·5372	34
28	·4476	32	28	·4770	32	28	·5071	32	28	·5377	32
30	·4481	30	30	·4775	30	30	·5076	30	30	·5383	30
32	·4485	28	32	·4780	28	32	·5081	28	32	·5388	28
34	·4490	26	34	·4785	26	34	·5086	26	34	·5393	26
36	·4495	24	36	·4790	24	36	·5091	24	36	·5398	24
38	·4500	22	38	·4795	22	38	·5096	22	38	·5403	22
40	·4505	20	40	·4800	20	40	·5101	20	40	·5408	20
42	·4510	18	42	·4805	18	42	·5106	18	42	·5414	18
44	·4515	16	44	·4810	16	44	·5111	16	44	·5419	16
46	·4520	14	46	·4815	14	46	·5116	14	46	·5424	14
48	·4524	12	48	·4820	12	48	·5121	12	48	·5429	12
50	·4529	10	50	·4825	10	50	·5126	10	50	·5434	10
52	·4534	8	52	·4830	8	52	·5132	8	52	·5439	8
54	·4539	6	54	·4835	6	54	·5137	6	54	·5445	6
56	·4544	4	56	·4840	4	56	·5142	4	56	·5450	4
58	·4549	2	58	·4845	2	58	·5147	2	58	·5455	2
60	·4554	**303** 0	60	·4850	**301** 0	60	·5152	**299** 0	60	·5460	**297** 0

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57 0	·4554	60	**59** 0	·4850	60	**61** 0	·5152	60	**63** 0	·5460	60
2	·4558	58	2	·4855	58	2	·5157	58	2	·5465	58
4	·4563	56	4	·4860	56	4	·5162	56	4	·5470	56
6	·4568	54	6	·4865	54	6	·5167	54	6	·5476	54
8	·4573	52	8	·4870	52	8	·5172	52	8	·5481	52
10	·4578	50	10	·4875	50	10	·5177	50	10	·5486	50
12	·4583	48	12	·4880	48	12	·5182	48	12	·5491	48
14	·4588	46	14	·4885	46	14	·5188	46	14	·5496	46
16	·4593	44	16	4890	44	16	·5193	44	16	·5502	44
18	·4598	42	18	·4895	42	18	·5198	42	18	·5507	42
20	·4602	40	20	·4900	40	20	·5203	40	20	·5512	40
22	·4607	38	22	·4905	38	22	·5208	38	22	·5517	38
24	·4612	36	24	·4910	36	24	·5213	36	24	·5522	36
26	·4617	34	26	·4915	34	26	·5218	34	26	·5528	34
28	·4622	32	28	·4920	32	28	·5223	32	28	·5533	32
30	·4627	30	30	·4925	30	30	·5228	30	30	·5538	30
32	·4632	28	32	·4930	28	32	·5234	28	32	·5543	28
34	·4637	26	34	·4935	26	34	·5239	26	34	·5548	26
36	·4642	24	36	·4940	24	36	·5244	24	36	·5554	24
38	·4647	22	38	·4945	22	38	·5249	22	38	·5559	22
40	·4652	20	40	·4950	20	40	·5254	20	40	·5564	20
42	·4656	18	42	·4955	18	42	·5259	18	42	·5569	18
44	·4661	16	44	·4960	16	44	·5264	16	44	·5575	16
46	·4666	14	46	·4965	14	46	·5269	14	46	·5580	14
48	·4671	12	48	·4970	12	48	·5274	12	48	·5585	12
50	·4676	10	50	·4975	10	50	·5280	10	50	·5590	10
52	·4681	8	52	·4980	8	52	·5285	8	52	·5595	8
54	·4686	6	54	·4985	6	54	·5290	6	54	·5601	6
56	·4691	4	56	·4990	4	56	·5295	4	56	·5606	4
58	·4696	2	58	·4995	2	58	·5300	2	58	·5611	2
60	·4701	**302** 0	60	·5000	**300** 0	60	·5305	**298** 0	60	·5615	**296** 0

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64 0	·5616	60	**66** 0	·5933	60	**68** 0	·6254	60	**70** 0	·6580	60
2	·5622	58	2	·5938	58	2	·6259	58	2	·6585	58
4	·5627	56	4	·5943	56	4	·6265	56	4	·6591	56
6	·5632	54	6	·5949	54	6	·6270	54	6	·6596	54
8	·5637	52	8	·5954	52	8	·6276	52	8	·6602	52
10	·5642	50	10	·5959	50	10	·6281	50	10	·6607	50
12	·5648	48	12	·5965	48	12	·6286	48	12	·6613	48
14	·5653	46	14	·5970	46	14	·6292	46	14	·6618	46
16	·5658	44	16	·5975	44	16	·6297	44	16	·6624	44
18	·5663	42	18	·5981	42	18	·6303	42	18	·6629	42
20	·5669	40	20	·5986	40	20	·6308	40	20	·6635	40
22	·5674	38	22	·5991	38	22	·6313	38	22	·6640	38
24	·5679	36	24	·5997	36	24	·6319	36	24	·6645	36
26	·5684	34	26	·6002	34	26	·6324	34	26	·6651	34
28	·5690	32	28	·6007	32	28	·6330	32	28	·6656	32
30	·5695	30	30	·6013	30	30	·6335	30	30	·6662	30
32	·5700	28	32	·6018	28	32	·6340	28	32	·6667	28
34	·5705	26	34	·6023	26	34	·6346	26	34	·6673	26
36	·5711	24	36	·6029	24	36	·6351	24	36	·6678	24
38	·5716	22	38	·6034	22	38	·6357	22	38	·6684	22
40	·5721	20	40	·6039	20	40	·6362	20	40	·6689	20
42	·5726	18	42	·6045	18	42	·6367	18	42	·6695	18
44	·5732	16	44	·6050	16	44	·6373	16	44	·6700	16
46	·5737	14	46	·6055	14	46	·6378	14	46	·6706	14
48	·5742	12	48	·6061	12	48	·6384	12	48	·6711	12
50	·5747	10	50	·6066	10	50	·6389	10	50	·6717	10
52	·5753	8	52	·6071	8	52	·6395	8	52	·6722	8
54	·5758	6	54	·6077	6	54	·6400	6	54	·6728	6
56	·5763	4	56	·6082	4	56	·6405	4	56	·6733	4
58	·5769	2	58	·6087	2	58	·6411	2	58	·6739	2
60	·5774	**295** 0	60	·6093	**293** 0	60	·6416	**291** 0	60	·6744	**289** 0

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65 0	·5774	60	**67** 0	·6093	60	**69** 0	·6416	60	**71** 0	·6744	60
2	·5779	58	2	·6098	58	2	·6422	58	2	·6750	58
4	·5784	56	4	·6103	56	4	·6427	56	4	·6755	56
6	·5790	54	6	·6109	54	6	·6433	54	6	·6761	54
8	·5795	52	8	·6114	52	8	·6438	52	8	·6766	52
10	·5800	50	10	·6119	50	10	·6443	50	10	·6772	50
12	·5805	48	12	·6125	48	12	·6449	48	12	·6777	48
14	·5811	46	14	·6130	46	14	·6454	46	14	·6783	46
16	·5816	44	16	·6136	44	16	·6460	44	16	·6788	44
18	·5821	42	18	·6141	42	18	·6465	42	18	·6794	42
20	·5827	40	20	·6146	40	20	·6471	40	20	·6799	40
22	·5832	38	22	·6152	38	22	·6476	38	22	·6805	38
24	·5837	36	24	·6157	36	24	·6482	36	24	·6810	36
26	·5842	34	26	·6162	34	26	·6487	34	26	·6816	34
28	·5848	32	28	·6168	32	28	·6492	32	28	·6821	32
30	·5853	30	30	·6173	30	30	·6498	30	30	·6827	30
32	·5858	28	32	·6179	28	32	·6503	28	32	·6832	28
34	·5864	26	34	·6184	26	34	·6509	26	34	·6838	26
36	·5869	24	36	·6189	24	36	·6514	24	36	·6844	24
38	·5874	22	38	·6195	22	38	·6520	22	38	·6849	22
40	·5880	20	40	·6200	20	40	·6525	20	40	·6855	20
42	·5885	18	42	·6205	18	42	·6531	18	42	·6860	18
44	·5890	16	44	·6211	16	44	·6536	16	44	·6866	16
46	·5895	14	46	·6216	14	46	·6542	14	46	·6871	14
48	·5901	12	48	·6222	12	48	·6547	12	48	·6877	12
50	·5906	10	50	·6227	10	50	·6552	10	50	·6882	10
52	·5911	8	52	·6232	8	52	·6558	8	52	·6888	8
54	·5917	6	54	·6238	6	54	·6563	6	54	·6893	6
56	·5922	4	56	·6243	4	56	·6569	4	56	·6899	4
58	·5927	2	58	·6249	2	58	·6574	2	58	·6904	2
60	·5933	**294** 0	60	·6254	**292** 0	60	·6580	**290** 0	60	·6910	**288** 0

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72 0	·6910	60	**74** 0	·7244	60	**76** 0	·7581	60	**78** 0	·7921	60
2	·6915	58	2	·7249	58	2	·7586	58	2	·7927	58
4	·6921	56	4	·7255	56	4	·7592	56	4	·7932	56
6	·6926	54	6	·7260	54	6	·7598	54	6	·7938	54
8	·6932	52	8	·7266	52	8	·7603	52	8	·7944	52
10	·6938	50	10	·7272	50	10	·7609	50	10	·7949	50
12	·6943	48	12	·7277	48	12	·7615	48	12	·7955	48
14	·6949	46	14	·7283	46	14	·7620	46	14	·7961	46
16	·6954	44	16	·7288	44	16	·7626	44	16	·7966	44
18	·6960	42	18	·7294	42	18	·7632	42	18	·7972	42
20	·6965	40	20	·7300	40	20	·7637	40	20	·7978	40
22	·6971	38	22	·7305	38	22	·7643	38	22	·7984	38
24	·6976	36	24	·7311	36	24	·7649	36	24	·7989	36
26	6982	34	26	·7316	34	26	·7654	34	26	·7995	34
28	·6987	32	28	·7322	32	28	·7660	32	28	·8001	32
30	·6993	30	30	·7328	30	30	·7666	30	30	·8006	30
32	·6998	28	32	·7333	28	32	·7671	28	32	·8012	28
34	·7004	26	34	·7339	26	34	·7677	26	34	·8018	26
36	·7010	24	36	·7344	24	36	·7683	24	36	·8023	24
38	·7015	22	38	·7350	22	38	·7688	22	38	·8029	22
40	·7021	20	40	·7356	20	40	·7694	20	40	·8035	20
42	·7026	18	42	·7361	18	42	·7700	18	42	·8041	18
44	·7032	16	44	·7367	16	44	·7705	16	44	·8046	16
46	·7037	14	46	·7372	14	46	·7711	14	46	·8052	14
48	·7043	12	48	·7378	12	48	·7716	12	48	·8058	12
50	·7048	10	50	·7384	10	50	·7722	10	50	·8063	10
52	·7054	8	52	·7389	8	52	·7728	8	52	·8069	8
54	·7060	6	54	·7395	6	54	·7733	6	54	·8075	6
56	·7065	4	56	·7401	4	56	·7739	4	56	·8080	4
58	·7071	2	58	·7406	2	58	·7745	2	58	·8086	2
60	·7076	**287** 0	60	·7412	**285** 0	60	·7750	**283** 0	60	·8092	**281** 0
° ′		° ′	° ′		° ′	° ′		° ′	° ′		° ′
73 0	·7076	60	**75** 0	·7412	60	**77** 0	·7750	60	**79** 0	·8092	60
2	·7082	58	2	·7417	58	2	·7756	58	2	·8098	58
4	·7087	56	4	·7423	56	4	·7762	56	4	·8103	56
6	·7093	54	6	·7429	54	6	·7767	54	6	·8109	54
8	·7099	52	8	·7434	52	8	·7773	52	8	·8115	52
10	·7104	50	10	·7440	50	10	·7779	50	10	·8120	50
12	·7110	48	12	·7446	48	12	·7785	48	12	·8126	48
14	·7115	46	14	·7451	46	14	·7790	46	14	·8132	46
16	·7121	44	16	·7457	44	16	·7796	44	16	·8138	44
18	·7126	42	18	·7462	42	18	·7802	42	18	·8143	42
20	·7132	40	20	·7468	40	20	·7807	40	20	·8149	40
22	·7138	38	22	·7474	38	22	·7813	38	22	·8155	38
24	·7143	36	24	·7479	36	24	·7819	36	24	·8160	36
26	·7149	34	26	·7485	34	26	·7824	34	26	·8166	34
28	·7154	32	28	·7491	32	28	·7830	32	28	·8172	32
30	·7160	30	30	·7496	30	30	·7836	30	30	·8178	30
32	·7165	28	32	·7502	28	32	·7841	28	32	·8183	28
34	·7171	26	34	·7507	26	34	·7847	26	34	·8189	26
36	·7177	24	36	·7513	24	36	·7853	24	36	·8195	24
38	·7182	22	38	·7519	22	38	·7858	22	38	·8201	22
40	·7188	20	40	·7524	20	40	·7864	20	40	·8206	20
42	·7193	18	42	·7530	18	42	·7870	18	42	·8212	18
44	·7199	16	44	·7536	16	44	·7875	16	44	·8218	16
46	·7205	14	46	·7541	14	46	·7881	14	46	·8223	14
48	·7210	12	48	·7547	12	48	·7887	12	48	·8229	12
50	·7216	10	50	·7553	10	50	·7892	10	50	·8235	10
52	·7221	8	52	·7558	8	52	·7898	8	52	·8241	8
54	·7227	6	54	·7564	6	54	·7904	6	54	·8246	6
56	·7232	4	56	·7569	4	56	·7910	4	56	·8252	4
58	·7238	2	58	·7575	2	58	·7915	2	58	·8258	2
60	·7244	**286** 0	60	·7581	**284** 0	60	·7921	**282** 0	60	·8264	**280** 0

° ′		° ′	° ′		° ′	° ′		° ′	° ′		° ′
96 0	1·1045	60	**98** 0	1·1392	60	**100** 0	1·1736	60	**102** 0	1·2079	60
2	1·1051	58	2	1·1397	58	2	1·1742	58	2	1·2085	58
4	1·1057	56	4	1·1403	56	4	1·1748	56	4	1·2090	56
6	1·1063	54	6	1·1409	54	6	1·1754	54	6	1·2096	54
8	1·1068	52	8	1·1415	52	8	1·1759	52	8	1·2102	52
10	1·1074	50	10	1·1421	50	10	1·1765	50	10	1·2108	50
12	1·1080	48	12	1·1426	48	12	1·1771	48	12	1·2113	48
14	1·1086	46	14	1·1432	46	14	1·1777	46	14	1·2119	46
16	1·1092	44	16	1·1438	44	16	1·1782	44	16	1·2125	44
18	1·1097	42	18	1·1444	42	18	1·1788	42	18	1·2130	42
20	1·1103	40	20	1·1449	40	20	1·1794	40	20	1·2136	40
22	1·1109	38	22	1·1455	38	22	1·1799	38	22	1·2142	38
24	1·1115	36	24	1·1461	36	24	1·1805	36	24	1·2147	36
26	1·1120	34	26	1·1467	34	26	1·1811	34	26	1·2153	34
28	1·1126	32	28	1·1472	32	28	1·1817	32	28	1·2159	32
30	1·1132	30	30	1·1478	30	30	1·1822	30	30	1·2164	30
32	1·1138	28	32	1·1484	28	32	1·1828	28	32	1·2170	28
34	1·1144	26	34	1·1490	26	34	1·1834	26	34	1·2176	26
36	1·1149	24	36	1·1495	24	36	1·1840	24	36	1·2181	24
38	1·1155	22	38	1·1501	22	38	1·1845	22	38	1·2187	22
40	1·1161	20	40	1·1507	20	40	1·1851	20	40	1·2193	20
42	1·1167	18	42	1·1513	18	42	1·1857	18	42	1·2198	18
44	1·1172	16	44	1·1518	16	44	1·1862	16	44	1·2204	16
46	1·1178	14	46	1·1524	14	46	1·1868	14	46	1·2210	14
48	1·1184	12	48	1·1530	12	48	1·1874	12	48	1·2215	12
50	1·1190	10	50	1·1536	10	50	1·1880	10	50	1·2221	10
52	1·1196	8	52	1·1541	8	52	1·1885	8	52	1·2227	8
54	1·1201	6	54	1·1547	6	54	1·1891	6	54	1·2233	6
56	1·1207	4	56	1·1553	4	56	1·1897	4	56	1·2238	4
58	1·1213	2	58	1·1559	2	58	1·1902	2	58	1·2244	2
60	1·1219	**263** 0	60	1·1564	**261** 0	60	1·1908	**259** 0	60	1·2250	**257** 0
° ′		° ′	° ′		° ′	° ′		° ′	° ′		° ′
97 0	1·1219	60	**99** 0	1·1564	60	**101** 0	1·1908	60	**103** 0	1·2250	60
2	1·1224	58	2	1·1570	58	2	1·1914	58	2	1·2255	58
4	1·1230	56	4	1·1576	56	4	1·1920	56	4	1·2261	56
6	1·1236	54	6	1·1582	54	6	1·1925	54	6	1·2267	54
8	1·1242	52	8	1·1587	52	8	1·1931	52	8	1·2272	52
10	1·1248	50	10	1·1593	50	10	1·1937	50	10	1·2278	50
12	1·1253	48	12	1·1599	48	12	1·1942	48	12	1·2284	48
14	1·1259	46	14	1·1605	46	14	1·1948	46	14	1·2289	46
16	1·1265	44	16	1·1610	44	16	1·1954	44	16	1·2295	44
18	1·1271	42	18	1·1616	42	18	1·1959	42	18	1·2300	42
20	1·1276	40	20	1·1622	40	20	1·1965	40	20	1·2306	40
22	1·1282	38	22	1·1628	38	22	1·1971	38	22	1·2312	38
24	1·1288	36	24	1·1633	36	24	1·1977	36	24	1·2317	36
26	1·1294	34	26	1·1639	34	26	1·1982	34	26	1·2323	34
28	1·1299	32	28	1·1645	32	28	1·1988	32	28	1·2329	32
30	1·1305	30	30	1·1650	30	30	1·1994	30	30	1·2334	30
32	1·1311	28	32	1·1656	28	32	1·1999	28	32	1·2340	28
34	1·1317	26	34	1·1662	26	34	1·2005	26	34	1·2346	26
36	1·1323	24	36	1·1668	24	36	1·2011	24	36	1·2351	24
38	1·1328	22	38	1·1673	22	38	1·2016	22	38	1·2357	22
40	1·1334	20	40	1·1679	20	40	1·2022	20	40	1·2363	20
42	1·1340	18	42	1·1685	18	42	1·2028	18	42	1·2368	18
44	1·1346	16	44	1·1691	16	44	1·2034	16	44	1·2374	16
46	1·1351	14	46	1·1696	14	46	1·2039	14	46	1·2380	14
48	1·1357	12	48	1·1702	12	48	1·2045	12	48	1·2385	12
50	1·1363	10	50	1·1708	10	50	1·2051	10	50	1·2391	10
52	1·1369	8	52	1·1714	8	52	1·2056	8	52	1·2397	8
54	1·1374	6	54	1·1719	6	54	1·2062	6	54	1·2402	6
56	1·1380	4	56	1·1725	4	56	1·2068	4	56	1·2408	4
58	1·1386	2	58	1·1731	2	58	1·2073	2	58	1·2414	2
60	1·1392	**262** 0	60	1·1736	**260** 0	60	1·2079	**258** 0	60	1·2419	**256** 0

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104 0	1·2419	60	**106** 0	1·2756	60	**108** 0	1·3090	60	**110** 0	1·3420	60
2	1·2425	58	2	1·2762	58	2	1·3096	58	2	1·3426	58
4	1·2431	56	4	1·2768	56	4	1·3101	56	4	1·3431	56
6	1·2436	54	6	1·2773	54	6	1·3107	54	6	1·3437	54
8	1·2442	52	8	1·2779	52	8	1·3112	52	8	1·3442	52
10	1·2447	50	10	1·2784	50	10	1·3118	50	10	1·3448	50
12	1·2453	48	12	1·2790	48	12	1·3123	48	12	1·3453	48
14	1·2459	46	14	1·2795	46	14	1·3129	46	14	1·3458	46
16	1·2464	44	16	1·2801	44	16	1·3134	44	16	1·3464	44
18	1·2470	42	18	1·2807	42	18	1·3140	42	18	1·3469	42
20	1·2476	40	20	1·2812	40	20	1·3145	40	20	1·3475	40
22	1·2481	38	22	1·2818	38	22	1·3151	38	22	1·3480	38
24	1·2487	36	24	1·2823	36	24	1·3156	36	24	1·3486	36
26	1·2493	34	26	1·2829	34	26	1·3162	34	26	1·3491	34
28	1·2498	32	28	1·2835	32	28	1·3168	32	28	1·3497	32
30	1·2504	30	30	1·2840	30	30	1·3173	30	30	1·3502	30
32	1·2509	28	32	1·2846	28	32	1·3179	28	32	1·3508	28
34	1·2515	26	34	1·2851	26	34	1·3184	26	34	1·3513	26
36	1·2521	24	36	1·2857	24	36	1·3190	24	36	1·3518	24
38	1·2526	22	38	1·2862	22	38	1·3195	22	38	1·3524	22
40	1·2532	20	40	1·2868	20	40	1·3201	20	40	1·3529	20
42	1·2538	18	42	1·2874	18	42	1·3206	18	42	1·3535	18
44	1·2543	16	44	1·2879	16	44	1·3212	16	44	1·3540	16
46	1·2549	14	46	1·2885	14	46	1·3217	14	46	1·3546	14
48	1·2554	12	48	1·2890	12	48	1·3223	12	48	1·3551	12
50	1·2560	10	50	1·2896	10	50	1·3228	10	50	1·3557	10
52	1·2566	8	52	1·2901	8	52	1·3234	8	52	1·3562	8
54	1·2571	6	54	1·2907	6	54	1·3239	6	54	1·3567	6
56	1·2577	4	56	1·2913	4	56	1·3245	4	56	1·3573	4
58	1·2583	2	58	1·2918	2	58	1·3250	2	58	1·3578	2
60	1·2588	**255** 0	60	1·2924	**253** 0	60	1·3256	**251** 0	60	1·3584	**249** 0
° ′		° ′	° ′		° ′	° ′		° ′	° ′		° ′
105 0	1·2588	60	**107** 0	1·2924	60	**109** 0	1·3256	60	**111** 0	1·3584	60
2	1·2594	58	2	1·2929	58	2	1·3261	58	2	1·3589	58
4	1·2599	56	4	1·2935	56	4	1·3267	56	4	1·3595	56
6	1·2605	54	6	1·2940	54	6	1·3272	54	6	1·3600	54
8	1·2611	52	8	1·2946	52	8	1·3278	52	8	1·3605	52
10	1·2616	50	10	1·2952	50	10	1·3283	50	10	1·3611	50
12	1·2622	48	12	1·2957	48	12	1·3289	48	12	1·3616	48
14	1·2628	46	14	1·2963	46	14	1·3294	46	14	1·3622	46
16	1·2633	44	16	1·2968	44	16	1·3300	44	16	1·3627	44
18	1·2639	42	18	1·2974	42	18	1·3305	42	18	1·3633	42
20	1·2644	40	20	1·2979	40	20	1·3311	40	20	1·3638	40
22	1·2650	38	22	1·2985	38	22	1·3316	38	22	1·3643	38
24	1·2656	36	24	1·2990	36	24	1·3322	36	24	1·3649	36
26	1·2661	34	26	1·2996	34	26	1·3327	34	26	1·3654	34
28	1·2667	32	28	1·3002	32	28	1·3333	32	28	1·3660	32
30	1·2672	30	30	1·3007	30	30	1·3338	30	30	1·3665	30
32	1·2678	28	32	1·3013	28	32	1·3344	28	32	1·3670	28
34	1·2684	26	34	1·3018	26	34	1·3349	26	34	1·3676	26
36	1·2689	24	36	1·3024	24	36	1·3355	24	36	1·3681	24
38	1·2695	22	38	1·3029	22	38	1·3360	22	38	1·3687	22
40	1·2700	20	40	1·3035	20	40	1·3365	20	40	1·3692	20
42	1·2706	18	42	1·3040	18	42	1·3371	18	42	1·3697	18
44	1·2712	16	44	1·3046	16	44	1·3376	16	44	1·3703	16
46	1·2717	14	46	1·3051	14	46	1·3382	14	46	1·3708	14
48	1·2723	12	48	1·3057	12	48	1·3387	12	48	1·3714	12
50	1·2728	10	50	1·3062	10	50	1·3393	10	50	1·3719	10
52	1·2734	8	52	1·3068	8	52	1·3398	8	52	1·3724	8
54	1·2740	6	54	1·3074	6	54	1·3404	6	54	1·3730	6
56	1·2745	4	56	1·3079	4	56	1·3409	4	56	1·3735	4
58	1·2751	2	58	1·3085	2	58	1·3415	2	58	1·3741	2
60	1·2756	**254** 0	60	1·3090	**252** 0	60	1·3420	**250** 0	60	1·3746	**248** 0

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112 0	1·3746	60	**114** 0	1·4067	60	**116** 0	1·4384	60	**118** 0	1·4695	60
2	1·3751	58	2	1·4073	58	2	1·4389	58	2	1·4700	58
4	1·3757	56	4	1·4078	56	4	1·4394	56	4	1·4705	56
6	1·3762	54	6	1·4083	54	6	1·4399	54	6	1·4710	54
8	1·3768	52	8	1·4089	52	8	1·4405	52	8	1·4715	52
10	1·3773	50	10	1·4094	50	10	1·4410	50	10	1·4720	50
12	1·3778	48	12	1·4099	48	12	1·4415	48	12	1·4726	48
14	1·3784	46	14	1·4105	46	14	1·4420	46	14	1·4731	46
16	1·3789	44	16	1·4110	44	16	1·4425	44	16	1·4736	44
18	1·3795	42	18	1·4115	42	18	1·4431	42	18	1·4741	42
20	1·3800	40	20	1·4120	40	20	1·4436	40	20	1·4746	40
22	1·3805	38	22	1·4126	38	22	1·4441	38	22	1·4751	38
24	1·3811	36	24	1·4131	36	24	1·4446	36	24	1·4756	36
26	1·3816	34	26	1·4136	34	26	1·4452	34	26	1·4761	34
28	1·3821	32	28	1·4142	32	28	1·4457	32	28	1·4766	32
30	1·3827	30	30	1·4147	30	30	1·4462	30	30	1·4772	30
32	1·3832	28	32	1·4152	28	32	1·4467	28	32	1·4777	28
34	1·3838	26	34	1·4158	26	34	1·4472	26	34	1·4782	26
36	1·3843	24	36	1·4163	24	36	1·4478	24	36	1·4787	24
38	1·3848	22	38	1·4168	22	38	1·4483	22	38	1·4792	22
40	1·3854	20	40	1·4173	20	40	1·4488	20	40	1·4797	20
42	1·3859	18	42	1·4179	18	42	1·4493	18	42	1·4802	18
44	1·3864	16	44	1·4184	16	44	1·4498	16	44	1·4807	16
46	1·3870	14	46	1·4189	14	46	1·4504	14	46	1·4812	14
48	1·3875	12	48	1·4195	12	48	1·4509	12	48	1·4818	12
50	1·3881	10	50	1·4200	10	50	1·4514	10	50	1·4823	10
52	1·3886	8	52	1·4205	8	52	1·4519	8	52	1·4828	8
54	1·3891	6	54	1·4210	6	54	1·4524	6	54	1·4833	6
56	1·3897	4	56	1·4216	4	56	1·4530	4	56	1·4838	4
58	1·3902	2	58	1·4221	2	58	1·4535	2	58	1·4843	2
60	1·3907	**247** 0	60	1·4226	**245** 0	60	1·4540	**243** 0	60	1·4848	**241** 0
° ′		° ′	° ′		° ′	° ′		° ′	° ′		° ′
113 0	1·3907	60	**115** 0	1·4226	60	**117** 0	1·4540	60	**119** 0	1·4848	60
2	1·3913	58	2	1·4231	58	2	1·4545	58	2	1·4853	58
4	1·3918	56	4	1·4237	56	4	1·4550	56	4	1·4858	56
6	1·3923	54	6	1·4242	54	6	1·4555	54	6	1·4863	54
8	1·3929	52	8	1·4247	52	8	1·4561	52	8	1·4868	52
10	1·3934	50	10	1·4253	50	10	1·4566	50	10	1·4874	50
12	1·3939	48	12	1·4258	48	12	1·4571	48	12	1·4879	48
14	1·3945	46	14	1·4263	46	14	1·4576	46	14	1·4884	46
16	1·3950	44	16	1·4268	44	16	1·4581	44	16	1·4889	44
18	1·3955	42	18	1·4274	42	18	1·4586	42	18	1·4894	42
20	1·3961	40	20	1·4279	40	20	1·4592	40	20	1·4899	40
22	1·3966	38	22	1·4284	38	22	1·4597	38	22	1·4904	38
24	1·3971	36	24	1·4289	36	24	1·4602	36	24	1·4909	36
26	1·3977	34	26	1·4295	34	26	1·4607	34	26	1·4914	34
28	1·3982	32	28	1·4300	32	28	1·4612	32	28	1·4919	32
30	1·3987	30	30	1·4305	30	30	1·4617	30	30	1·4924	30
32	1·3993	28	32	1·4310	28	32	1·4623	28	32	1·4929	28
34	1·3998	26	34	1·4316	26	34	1·4628	26	34	1·4934	26
36	1·4003	24	36	1·4321	24	36	1·4633	24	36	1·4939	24
38	1·4009	22	38	1·4326	22	38	1·4638	22	38	1·4944	22
40	1·4014	20	40	1·4331	20	40	1·4643	20	40	1·4950	20
42	1·4019	18	42	1·4337	18	42	1·4648	18	42	1·4955	18
44	1·4025	16	44	1·4342	16	44	1·4654	16	44	1·4960	16
46	1·4030	14	46	1·4347	14	46	1·4659	14	46	1·4965	14
48	1·4035	12	48	1·4352	12	48	1·4664	12	48	1·4970	12
50	1·4041	10	50	1·4358	10	50	1·4669	10	50	1·4975	10
52	1·4046	8	52	1·4363	8	52	1·4674	8	52	1·4980	8
54	1·4051	6	54	1·4368	6	54	1·4679	6	54	1·4985	6
56	1·4057	4	56	1·4373	4	56	1·4684	4	56	1·4990	4
58	1·4062	2	58	1·4378	2	58	1·4690	2	58	1·4995	2
60	1·4067	**246** 0	60	1·4384	**244** 0	60	1·4695	**242** 0	60	1·5000	**240** 0

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120 0	1·5000	60	**122** 0	1·5299	60	**124** 0	1·5592	60	**126** 0	1·5878	60
2	1·5005	58	2	1·5304	58	2	1·5597	58	2	1·5883	58
4	1·5010	56	4	1·5309	56	4	1·5602	56	4	1·5887	56
6	1·5015	54	6	1·5314	54	6	1·5606	54	6	1·5892	54
8	1·5020	52	8	1·5319	52	8	1·5611	52	8	1·5897	52
10	1·5025	50	10	1·5324	50	10	1·5616	50	10	1·5901	50
12	1·5030	48	12	1·5329	48	12	1·5621	48	12	1·5906	48
14	1·5035	46	14	1·5334	46	14	1·5626	46	14	1·5911	46
16	1·5040	44	16	1·5339	44	16	1·5630	44	16	1·5915	44
18	1·5045	42	18	1·5344	42	18	1·5635	42	18	1·5920	42
20	1·5050	40	20	1·5348	40	20	1·5640	40	20	1·5925	40
22	1·5055	38	22	1·5353	38	22	1·5645	38	22	1·5930	38
24	1·5060	36	24	1·5358	36	24	1·5650	36	24	1·5934	36
26	1·5065	34	26	1·5363	34	26	1·5654	34	26	1·5939	34
28	1·5070	32	28	1·5368	32	28	1·5659	32	28	1·5944	32
30	1·5075	30	30	1·5373	30	30	1·5664	30	30	1·5948	30
32	1·5080	28	32	1·5378	28	32	1·5669	28	32	1·5953	28
34	1·5085	26	34	1·5383	26	34	1·5674	26	34	1·5958	26
36	1·5090	24	36	1·5388	24	36	1·5678	24	36	1·5962	24
38	1·5095	22	38	1·5393	22	38	1·5683	22	38	1·5967	22
40	1·5100	20	40	1·5398	20	40	1·5688	20	40	1·5972	20
42	1·5105	18	42	1·5402	18	42	1·5693	18	42	1·5976	18
44	1·5110	16	44	1·5407	16	44	1·5698	16	44	1·5981	16
46	1·5115	14	46	1·5412	14	46	1·5702	14	46	1·5986	14
48	1·5120	12	48	1·5417	12	48	1·5707	12	48	1·5990	12
50	1·5125	10	50	1·5422	10	50	1·5712	10	50	1·5995	10
52	1·5130	8	52	1·5427	8	52	1·5717	8	52	1·6000	8
54	1·5135	6	54	1·5432	6	54	1·5721	6	54	1·6004	6
56	1·5140	4	56	1·5437	4	56	1·5726	4	56	1·6009	4
58	1·5145	2	58	1·5442	2	58	1·5731	2	58	1·6014	2
60	1·5150	**239** 0	60	1·5446	**237** 0	60	1·5736	**235** 0	60	1·6018	**233** 0

° ′		° ′	° ′		° ′	° ′		° ′	° ′		° ′
121 0	1·5150	60	**123** 0	1·5446	60	**125** 0	1·5736	60	**127** 0	1·6018	60
2	1·5155	58	2	1·5451	58	2	1·5741	58	2	1·6023	58
4	1·5160	56	4	1·5456	56	4	1·5745	56	4	1·6027	56
6	1·5165	54	6	1·5461	54	6	1·5750	54	6	1·6032	54
8	1·5170	52	8	1·5466	52	8	1·5755	52	8	1·6037	52
10	1·5175	50	10	1·5471	50	10	1·5760	50	10	1·6041	50
12	1·5180	48	12	1·5476	48	12	1·5764	48	12	1·6046	48
14	1·5185	46	14	1·5480	46	14	1·5769	46	14	1·6051	46
16	1·5190	44	16	1·5485	44	16	1·5774	44	16	1·6055	44
18	1·5195	42	18	1·5490	42	18	1·5779	42	18	1·6060	42
20	1·5200	40	20	1·5495	40	20	1·5783	40	20	1·6065	40
22	1·5205	38	22	1·5500	38	22	1·5788	38	22	1 6069	38
24	1·5210	36	24	1·5505	36	24	1·5793	36	24	1·6074	36
26	1·5215	34	26	1·5510	34	26	1·5798	34	26	1·6078	34
28	1·5220	32	28	1·5515	32	28	1·5802	32	28	1·6083	32
30	1·5225	30	30	1·5519	30	30	1·5807	30	30	1·6088	30
32	1·5230	28	32	1·5524	28	32	1·5812	28	32	1·6092	28
34	1·5235	26	34	1·5529	26	34	1·5816	26	34	1·6097	26
36	1·5240	24	36	1·5534	24	36	1·5821	24	36	1·6101	24
38	1·5245	22	38	1·5539	22	38	1·5826	22	38	1·6106	22
40	1·5250	20	40	1·5544	20	40	1·5831	20	40	1·6111	20
42	1·5255	18	42	1·5548	18	42	1·5835	18	42	1·6115	18
44	1·5260	16	44	1·5553	16	44	1·5840	16	44	1·6120	16
46	1·5265	14	46	1·5558	14	46	1·5845	14	46	1·6124	14
48	1·5270	12	48	1·5563	12	48	1·5850	12	48	1·6129	12
50	1·5275	10	50	1·5568	10	50	1·5854	10	50	1·6134	10
52	1·5279	8	52	1·5573	8	52	1·5859	8	52	1·6138	8
54	1·5284	6	54	1·5577	6	54	1·5864	6	54	1·6143	6
56	1·5289	4	56	1·5582	4	56	1·5868	4	56	1·6147	4
58	1·5294	2	58	1·5587	2	58	1·5873	2	58	1·6152	2
60	1·5299	**238** 0	60	1·5592	**236** 0	60	1·5878	**234** 0	60	1·6157	**232** 0

TABLE OF VERSED SINES.

° ′		° ′	° ′		° ′	° ′		° ′	° ′		° ′
128 0	1·6157	60	**130** 0	1·6428	60	**132** 0	1·6691	60	**134** 0	1·6947	60
2	1·6161	58	2	1·6432	58	2	1·6696	58	2	1·6951	58
4	1·6166	56	4	1·6437	56	4	1·6700	56	4	1·6955	56
6	1·6170	54	6	1·6441	54	6	1·6704	54	6	1·6959	54
8	1·6175	52	8	1·6446	52	8	1·6709	52	8	1·6963	52
10	1·6180	50	10	1·6450	50	10	1·6713	50	10	1·6967	50
12	1·6184	48	12	1·6455	48	12	1·6717	48	12	1·6972	48
14	1·6189	46	14	1·6459	46	14	1·6722	46	14	1·6976	46
16	1·6193	44	16	1·6463	44	16	1·6726	44	16	1·6980	44
18	1·6198	42	18	1·6468	42	18	1·6730	42	18	1·6984	42
20	1·6202	40	20	1·6472	40	20	1·6734	40	20	1·6988	40
22	1·6207	38	22	1·6477	38	22	1·6739	38	22	1·6992	38
24	1·6211	36	24	1·6481	36	24	1·6743	36	24	1·6997	36
26	1·6216	34	26	1·6486	34	26	1·6747	34	26	1·7001	34
28	1·6221	32	28	1·6490	32	28	1·6752	32	28	1·7005	32
30	1·6225	30	30	1·6494	30	30	1·6756	30	30	1·7009	30
32	1·6230	28	32	1·6499	28	32	1·6760	28	32	1·7013	28
34	1·6234	26	34	1·6503	26	34	1·6764	26	34	1·7017	26
36	1·6239	24	36	1·6508	24	36	1·6769	24	36	1·7022	24
38	1·6243	22	38	1·6512	22	38	1·6773	22	38	1·7026	22
40	1·6248	20	40	1·6517	20	40	1·6777	20	40	1·7030	20
42	1·6252	18	42	1·6521	18	42	1·6782	18	42	1·7034	18
44	1·6257	16	44	1·6525	16	44	1·6786	16	44	1·7038	16
46	1·6262	14	46	1·6530	14	46	1·6790	14	46	1·7042	14
48	1·6266	12	48	1·6534	12	48	1·6794	12	48	1·7046	12
50	1·6271	10	50	1·6539	10	50	1·6799	10	50	1·7050	10
52	1·6275	8	52	1·6543	8	52	1·6803	8	52	1·7055	8
54	1·6280	6	54	1·6547	6	54	1·6807	6	54	1·7059	6
56	1·6284	4	56	1·6552	4	56	1·6811	4	56	1·7063	4
58	1·6289	2	58	1·6556	2	58	1·6816	2	58	1·7067	2
60	1·6293	**231** 0	60	1·6561	**229** 0	60	1·6820	**227** 0	60	1·7071	**225** 0

° ′		° ′	° ′		° ′	° ′		° ′	° ′		° ′
129 0	1·6293	60	**131** 0	1·6561	60	**133** 0	1·6820	60	**135** 0	1·7071	60
2	1·6298	58	2	1·6565	58	2	1·6824	58	2	1·7075	58
4	1·6302	56	4	1·6569	56	4	1·6828	56	4	1·7079	56
6	1·6307	54	6	1·6574	54	6	1·6833	54	6	1·7083	54
8	1·6311	52	8	1·6578	52	8	1·6837	52	8	1·7088	52
10	1·6316	50	10	1·6583	50	10	1·6841	50	10	1·7092	50
12	1·6320	48	12	1·6587	48	12	1·6845	48	12	1·7096	48
14	1·6325	46	14	1·6591	46	14	1·6850	46	14	1·7100	46
16	1·6329	44	16	1·6596	44	16	1·6854	44	16	1·7104	44
18	1·6334	42	18	1·6600	42	18	1·6858	42	18	1·7108	42
20	1·6338	40	20	1·6604	40	20	1·6862	40	20	1·7112	40
22	1·6343	38	22	1·6609	38	22	1·6867	38	22	1·7116	38
24	1·6347	36	24	1·6613	36	24	1·6871	36	24	1·7120	36
26	1·6352	34	26	1·6617	34	26	1·6875	34	26	1·7124	34
28	1·6356	32	28	1·6622	32	28	1·6879	32	28	1·7128	32
30	1·6361	30	30	1·6626	30	30	1·6884	30	30	1·7133	30
32	1·6365	28	32	1·6631	28	32	1·6888	28	32	1·7137	28
34	1·6370	26	34	1·6635	26	34	1·6892	26	34	1·7141	26
36	1·6374	24	36	1·6639	24	36	1·6896	24	36	1·7145	24
38	1·6379	22	38	1·6644	22	38	1·6900	22	38	1·7149	22
40	1·6383	20	40	1·6648	20	40	1·6905	20	40	1·7153	20
42	1·6388	18	42	1·6652	18	42	1·6909	18	42	1·7157	18
44	1·6392	16	44	1·6657	16	44	1·6913	16	44	1·7161	16
46	1·6397	14	46	1·6661	14	46	1·6917	14	46	1·7165	14
48	1·6401	12	48	1·6665	12	48	1·6921	12	48	1·7169	12
50	1·6406	10	50	1·6670	10	50	1·6926	10	50	1·7173	10
52	1·6410	8	52	1·6674	8	52	1·6930	8	52	1·7177	8
54	1·6414	6	54	1·6678	6	54	1·6934	6	54	1·7181	6
56	1·6419	4	56	1·6683	4	56	1·6938	4	56	1·7185	4
58	1·6423	2	58	1·6687	2	58	1·6942	2	58	1·7189	2
60	1·6428	**230** 0	60	1·6691	**228** 0	60	1·6947	**226** 0	60	1·7193	**224** 0

° ′		° ′	° ′		° ′	° ′		° ′	° ′		° ′
136 0	1·7193	60	**138** 0	1·7431	60	**140** 0	1·7660	60	**142** 0	1·7880	60
2	1·7197	58	2	1·7435	58	2	1·7664	58	2	1·7884	58
4	1·7201	56	4	1·7439	56	4	1·7668	56	4	1·7887	56
6	1·7206	54	6	1·7443	54	6	1·7672	54	6	1·7891	54
8	1·7210	52	8	1·7447	52	8	1·7675	52	8	1·7894	52
10	1·7214	50	10	1·7451	50	10	1·7679	50	10	1·7898	50
12	1·7218	48	12	1·7455	48	12	1·7683	48	12	1·7902	48
14	1·7222	46	14	1·7459	46	14	1·7687	46	14	1·7905	46
16	1·7226	44	16	1·7463	44	16	1·7690	44	16	1·7909	44
18	1·7230	42	18	1·7466	42	18	1·7694	42	18	1·7912	42
20	1·7234	40	20	1·7470	40	20	1·7698	40	20	1·7916	40
22	1·7238	38	22	1·7474	38	22	1·7701	38	22	1·7919	38
24	1·7242	36	24	1·7478	36	24	1·7705	36	24	1·7923	36
26	1·7246	34	26	1·7482	34	26	1·7709	34	26	1·7926	34
28	1·7250	32	28	1·7486	32	28	1·7713	32	28	1·7930	32
30	1·7254	30	30	1·7490	30	30	1·7716	30	30	1·7934	30
32	1·7258	28	32	1·7493	28	32	1·7720	28	32	1·7937	28
34	1·7262	26	34	1·7497	26	34	1·7724	26	34	1·7941	26
36	1·7266	24	36	1·7501	24	36	1·7727	24	36	1·7944	24
38	1·7270	22	38	1·7505	22	38	1·7731	22	38	1·7948	22
40	1·7274	20	40	1·7509	20	40	1·7735	20	40	1·7951	20
42	1·7278	18	42	1·7513	18	42	1·7738	18	42	1·7955	18
44	1·7282	16	44	1·7516	16	44	1·7742	16	44	1·7958	16
46	1·7286	14	46	1·7520	14	46	1·7746	14	46	1·7962	14
48	1·7290	12	48	1·7524	12	48	1·7749	12	48	1·7965	12
50	1·7294	10	50	1·7528	10	50	1·7753	10	50	1·7969	10
52	1·7298	8	52	1·7532	8	52	1·7757	8	52	1·7972	8
54	1·7302	6	54	1·7536	6	54	1·7760	6	54	1·7976	6
56	1·7306	4	56	1·7539	4	56	1·7764	4	56	1·7979	4
58	1·7310	2	58	1·7543	2	58	1·7768	2	58	1·7983	2
60	1·7314	**223** 0	60	1·7547	**221** 0	60	1·7771	**219** 0	60	1·7986	**217** 0

° ′		° ′	° ′		° ′	° ′		° ′	° ′		° ′
137 0	1·7314	60	**139** 0	1·7547	60	**141** 0	1·7771	60	**143** 0	1·7986	60
2	1·7318	58	2	1·7551	58	2	1·7775	58	2	1·7990	58
4	1·7321	56	4	1·7555	56	4	1·7779	56	4	1·7993	56
6	1·7325	54	6	1·7559	54	6	1·7782	54	6	1·7997	54
8	1·7329	52	8	1·7562	52	8	1·7786	52	8	1·8000	52
10	1·7333	50	10	1·7566	50	10	1·7790	50	10	1·8004	50
12	1·7337	48	12	1·7570	48	12	1·7793	48	12	1·8007	48
14	1·7341	46	14	1·7574	46	14	1·7797	46	14	1·8011	46
16	1·7345	44	16	1·7578	44	16	1·7801	44	16	1·8014	44
18	1·7349	42	18	1·7581	42	18	1·7804	42	18	1·8018	42
20	1·7353	40	20	1·7585	40	20	1·7808	40	20	1·8021	40
22	1·7357	38	22	1·7589	38	22	1·7812	38	22	1·8025	38
24	1·7361	36	24	1·7593	36	24	1·7815	36	24	1·8028	36
26	1·7365	34	26	1·7596	34	26	1·7819	34	26	1·8032	34
28	1·7369	32	28	1·7600	32	28	1·7822	32	28	1·8035	32
30	1·7373	30	30	1·7604	30	30	1·7826	30	30	1·8039	30
32	1·7377	28	32	1·7608	28	32	1·7830	28	32	1·8042	28
34	1·7381	26	34	1·7612	26	34	1·7833	26	34	1·8045	26
36	1·7385	24	36	1·7615	24	36	1·7837	24	36	1·8049	24
38	1·7388	22	38	1·7619	22	38	1·7841	22	38	1·8052	22
40	1·7392	20	40	1·7623	20	40	1·7844	20	40	1·8056	20
42	1·7396	18	42	1·7627	18	42	1·7848	18	42	1·8059	18
44	1·7400	16	44	1·7630	16	44	1·7851	16	44	1·8063	16
46	1·7404	14	46	1·7634	14	46	1·7855	14	46	1·8066	14
48	1·7408	12	48	1·7638	12	48	1·7859	12	48	1·8070	12
50	1·7412	10	50	1·7642	10	50	1·7862	10	50	1·8073	10
52	1·7416	8	52	1·7645	8	52	1·7866	8	52	1·8076	8
54	1·7420	6	54	1·7649	6	54	1·7869	6	54	1·8080	6
56	1·7424	4	56	1·7653	4	56	1·7873	4	56	1·8083	4
58	1·7428	2	58	1·7657	2	58	1·7877	2	58	1·8087	2
60	1·7431	**222** 0	60	1·7660	**220** 0	60	1·7880	**218** 0	60	1·8090	**216** 0

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144 0	1·8090	60	**146** 0	1·8290	60	**148** 0	1·8480	60	**150** 0	1·8660	60
2	1·8094	58	2	1·8294	58	2	1·8484	58	2	1·8663	58
4	1·8097	56	4	1·8297	56	4	1·8487	56	4	1·8666	56
6	1·8100	54	6	1·8300	54	6	1·8490	54	6	1·8669	54
8	1·8104	52	8	1·8303	52	8	1·8493	52	8	1·8672	52
10	1·8107	50	10	1·8307	50	10	1·8496	50	10	1·8675	50
12	1·8111	48	12	1·8310	48	12	1·8499	48	12	1·8678	48
14	1·8114	46	14	1·8313	46	14	1·8502	46	14	1·8681	46
16	1·8117	44	16	1·8316	44	16	1·8505	44	16	1·8683	44
18	1·8121	42	18	1·8320	42	18	1·8508	42	18	1·8686	42
20	1·8124	40	20	1·8323	40	20	1·8511	40	20	1·8689	40
22	1·8128	38	22	1·8326	38	22	1·8514	38	22	1·8692	38
24	1·8131	36	24	1·8329	36	24	1·8517	36	24	1·8695	36
26	1·8134	34	26	1·8332	34	26	1·8520	34	26	1·8698	34
28	1·8138	32	28	1·8336	32	28	1·8523	32	28	1·8701	32
30	1·8141	30	30	1·8339	30	30	1·8526	30	30	1·8704	30
32	1·8145	28	32	1·8342	28	32	1·8529	28	32	1·8706	28
34	1·8148	26	34	1·8345	26	34	1·8532	26	34	1·8709	26
36	1·8151	24	36	1·8348	24	36	1·8536	24	36	1·8712	24
38	1·8155	22	38	1·8352	22	38	1·8539	22	38	1·8715	22
40	1·8158	20	40	1·8355	20	40	1·8542	20	40	1·8718	20
42	1·8161	18	42	1·8358	18	42	1·8545	18	42	1·8721	18
44	1·8165	16	44	1·8361	16	44	1·8548	16	44	1·8724	16
46	1·8168	14	46	1·8364	14	46	1·8551	14	46	1·8726	14
48	1·8171	12	48	1·8368	12	48	1·8554	12	48	1·8729	12
50	1·8175	10	50	1·8371	10	50	1·8557	10	50	1·8732	10
52	1·8178	8	52	1·8374	8	52	1·8560	8	52	1·8735	8
54	1·8181	6	54	1·8377	6	54	1·8563	6	54	1·8738	6
56	1·8185	4	56	1·8380	4	56	1·8566	4	56	1·8741	4
58	1·8188	2	58	1·8384	2	58	1·8569	2	58	1·8743	2
60	1·8192	**215** 0	60	1·8387	**213** 0	60	1·8572	**211** 0	60	1·8746	**209** 0

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145 0	1·8192	60	**147** 0	1·8387	60	**149** 0	1·8572	60	**151** 0	1·8746	60
2	1·8195	58	2	1·8390	58	2	1·8575	58	2	1·8749	58
4	1·8198	56	4	1·8393	56	4	1·8578	56	4	1·8752	56
6	1·8202	54	6	1·8396	54	6	1·8581	54	6	1·8755	54
8	1·8205	52	8	1·8399	52	8	1·8584	52	8	1·8757	52
10	1·8208	50	10	1·8403	50	10	1·8587	50	10	1·8760	50
12	1·8211	48	12	1·8406	48	12	1·8590	48	12	1·8763	48
14	1·8215	46	14	1·8409	46	14	1·8593	46	14	1·8766	46
16	1·8218	44	16	1·8412	44	16	1·8596	44	16	1·8769	44
18	1·8221	42	18	1·8415	42	18	1·8599	42	18	1·8771	42
20	1·8225	40	20	1·8418	40	20	1·8601	40	20	1·8774	40
22	1·8228	38	22	1·8421	38	22	1·8604	38	22	1·8777	38
24	1·8231	36	24	1·8425	36	24	1·8607	36	24	1·8780	36
26	1·8235	34	26	1·8428	34	26	1·8610	34	26	1·8783	34
28	1·8238	32	28	1·8431	32	28	1·8613	32	28	1·8785	32
30	1·8241	30	30	1·8434	30	30	1·8616	30	30	1·8788	30
32	1·8245	28	32	1·8437	28	32	1·8619	28	32	1·8791	28
34	1·8248	26	34	1·8440	26	34	1·8622	26	34	1·8794	26
36	1·8251	24	36	1·8443	24	36	1·8625	24	36	1·8796	24
38	1·8254	22	38	1·8446	22	38	1·8628	22	38	1·8799	22
40	1·8258	20	40	1·8450	20	40	1·8631	20	40	1·8802	20
42	1·8261	18	42	1·8453	18	42	1·8634	18	42	1·8805	18
44	1·8264	16	44	1·8456	16	44	1·8637	16	44	1·8808	16
46	1·8268	14	46	1·8459	14	46	1·8640	14	46	1·8810	14
48	1·8271	12	48	1·8462	12	48	1·8643	12	48	1·8813	12
50	1·8274	10	50	1·8465	10	50	1·8646	10	50	1·8816	10
52	1·8277	8	52	1·8468	8	52	1·8649	8	52	1·8819	8
54	1·8281	6	54	1·8471	6	54	1·8652	6	54	1·8821	6
56	1·8284	4	56	1·8474	4	56	1·8654	4	56	1·8824	4
58	1·8287	2	58	1·8477	2	58	1·8657	2	58	1·8827	2
60	1·8290	**214** 0	60	1·8480	**212** 0	60	1·8660	**210** 0	60	1·8829	**208** 0

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152 0	1·8829	60	**154** 0	1·8988	60	**156** 0	1·9135	60	**158** 0	1·9272	60
2	1·8832	58	2	1·8990	58	2	1·9138	58	2	1·9274	58
4	1·8835	56	4	1·8993	56	4	1·9140	56	4	1·9276	56
6	1·8838	54	6	1·8996	54	6	1·9143	54	6	1·9278	54
8	1·8840	52	8	1·8998	52	8	1·9145	52	8	1·9281	52
10	1·8843	50	10	1·9001	50	10	1·9147	50	10	1·9283	50
12	1·8846	48	12	1·9003	48	12	1·9150	48	12	1·9285	48
14	1·8849	46	14	1·9006	46	14	1·9152	46	14	1·9287	46
16	1·8851	44	16	1·9008	44	16	1·9154	44	16	1·9289	44
18	1·8854	42	18	1·9011	42	18	1·9157	42	18	1·9291	42
20	1·8857	40	20	1·9013	40	20	1·9159	40	20	1·9293	40
22	1·8859	38	22	1·9016	38	22	1·9161	38	22	1·9296	38
24	1·8862	36	24	1·9018	36	24	1·9164	36	24	1·9298	36
26	1·8865	34	26	1·9021	34	26	1·9166	34	26	1·9300	34
28	1·8867	32	28	1·9023	32	28	1·9168	32	28	1·9302	32
30	1·8870	30	30	1·9026	30	30	1·9171	30	30	1·9304	30
32	1·8873	28	32	1·9028	28	32	1·9173	28	32	1·9306	28
34	1·8875	26	34	1·9031	26	34	1·9175	26	34	1·9308	26
36	1·8878	24	36	1·9033	24	36	1·9178	24	36	1·9311	24
38	1·8881	22	38	1·9036	22	38	1·9180	22	38	1·9313	22
40	1·8884	20	40	1·9038	20	40	1·9182	20	40	1·9315	20
42	1·8886	18	42	1·9041	18	42	1·9184	18	42	1·9317	18
44	1·8889	16	44	1·9043	16	44	1·9187	16	44	1·9319	16
46	1·8892	14	46	1·9046	14	46	1·9189	14	46	1·9321	14
48	1·8894	12	48	1·9048	12	48	1·9191	12	48	1·9323	12
50	1·8897	10	50	1·9051	10	50	1·9194	10	50	1·9325	10
52	1·8899	8	52	1·9053	8	52	1·9196	8	52	1·9327	8
54	1·8902	6	54	1·9056	6	54	1·9198	6	54	1·9330	6
56	1·8905	4	56	1·9058	4	56	1·9200	4	56	1·9332	4
58	1·8907	2	58	1·9061	2	58	1·9203	2	58	1·9334	2
60	1·8910	**207** 0	60	1·9063	**205** 0	60	1·9205	**203** 0	60	1·9336	**201** 0
° ′		° ′	° ′		° ′	° ′		° ′	° ′		° ′
153 0	1·8910	60	**155** 0	1·9063	60	**157** 0	1·9205	60	**159** 0	1·9336	60
2	1·8913	58	2	1·9066	58	2	1·9207	58	2	1·9338	58
4	1·8915	56	4	1·9068	56	4	1·9210	56	4	1·9340	56
6	1·8918	54	6	1·9070	54	6	1·9212	54	6	1·9342	54
8	1·8921	52	8	1·9073	52	8	1·9214	52	8	1·9344	52
10	1·8923	50	10	1·9075	50	10	1·9216	50	10	1·9346	50
12	1·8926	48	12	1·9078	48	12	1·9219	48	12	1·9348	48
14	1·8928	46	14	1·9080	46	14	1·9221	46	14	1·9350	46
16	1·8931	44	16	1·9083	44	16	1·9223	44	16	1·9352	44
18	1·8934	42	18	1·9085	42	18	1·9225	42	18	1·9354	42
20	1·8936	40	20	1·9088	40	20	1·9228	40	20	1·9356	40
22	1·8939	38	22	1·9090	38	22	1·9230	38	22	1·9359	38
24	1·8942	36	24	1·9092	36	24	1·9232	36	24	1·9361	36
26	1·8944	34	26	1·9095	34	26	1·9234	34	26	1·9363	34
28	1·8947	32	28	1·9097	32	28	1·9237	32	28	1·9365	32
30	1·8949	30	30	1·9100	30	30	1·9239	30	30	1·9367	30
32	1·8952	28	32	1·9102	28	32	1·9241	28	32	1·9369	28
34	1·8955	26	34	1·9104	26	34	1·9243	26	34	1·9371	26
36	1·8957	24	36	1·9107	24	36	1·9245	24	36	1·9373	24
38	1·8960	22	38	1·9109	22	38	1·9248	22	38	1·9375	22
40	1·8962	20	40	1·9112	20	40	1·9250	20	40	1·9377	20
42	1·8965	18	42	1·9114	18	42	1·9252	18	42	1·9379	18
44	1·8967	16	44	1·9116	16	44	1·9254	16	44	1·9381	16
46	1·8970	14	46	1·9119	14	46	1·9257	14	46	1·9383	14
48	1·8973	12	48	1·9121	12	48	1·9259	12	48	1·9385	12
50	1·8975	10	50	1·9124	10	50	1·9261	10	50	1·9387	10
52	1·8978	8	52	1·9126	8	52	1·9263	8	52	1·9389	8
54	1·8980	6	54	1·9128	6	54	1·9265	6	54	1·9391	6
56	1·8983	4	56	1·9131	4	56	1·9267	4	56	1·9393	4
58	1·8985	2	58	1·9133	2	58	1·9270	2	58	1·9395	2
60	1·8988	**206** 0	60	1·9135	**204** 0	60	1·9272	**202** 0	60	1·9397	**200** 0

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160 0	1·9397	60	**162** 0	1·9511	60	**164** 0	1·9613	60	**166** 0	1·9703	60
2	1·9399	58	2	1·9512	58	2	1·9614	58	2	1·9704	58
4	1·9401	56	4	1·9514	56	4	1·9616	56	4	1·9706	56
6	1·9403	54	6	1·9516	54	6	1·9617	54	6	1·9707	54
8	1·9405	52	8	1·9518	52	8	1·9619	52	8	1·9709	52
10	1·9407	59	10	1·9520	50	10	1·9621	50	10	1·9710	50
12	1·9409	48	12	1·9521	48	12	1·9622	48	12	1·9711	48
14	1·9411	46	14	1·9523	46	14	1·9624	46	14	1·9713	46
16	1·9413	44	16	1·9525	44	16	1·9625	44	16	1·9714	44
18	1·9415	42	18	1·9527	42	18	1·9627	42	18	1·9715	42
20	1·9417	40	20	1·9528	40	20	1·9628	40	20	1·9717	40
22	1·9419	38	22	1·9530	38	22	1·9630	38	22	1·9718	38
24	1·9421	36	24	1·9532	36	24	1·9632	36	24	1·9720	36
26	1·9423	34	26	1·9534	34	26	1·9633	34	26	1·9721	34
28	1·9424	32	28	1·9535	32	28	1·9635	32	28	1·9722	32
30	1·9426	30	30	1·9537	30	30	1·9636	30	30	1·9724	30
32	1·9428	28	32	1·9539	28	32	1·9638	28	32	1·9725	28
34	1·9430	26	34	1·9541	26	34	1·9639	26	34	1·9726	26
36	1·9432	24	36	1·9542	24	36	1·9641	24	36	1·9728	24
38	1·9434	22	38	1·9514	22	38	1·9642	22	38	1·9729	22
40	1·9436	20	40	1·9546	20	40	1·9644	20	40	1·9730	20
42	1·9438	18	42	1·9548	18	42	1·9646	18	42	1·9732	18
44	1·9440	16	44	1·9549	16	44	1·9647	16	44	1·9733	16
46	1·9442	14	46	1·9551	14	46	1·9649	14	46	1·9734	14
48	1·9444	12	48	1·9553	12	48	1·9650	12	48	1·9736	12
50	1·9446	10	50	1·9555	10	50	1·9652	10	50	1·9737	10
52	1·9448	8	52	1·9556	8	52	1·9653	8	52	1·9738	8
54	1·9449	6	54	1·9558	6	54	1·9655	6	54	1·9740	6
56	1·9451	4	56	1·9560	4	56	1·9656	4	56	1·9741	4
58	1·9453	2	58	1·9561	2	58	1·9658	2	58	1·9742	2
60	1·9455	**199** 0	60	1·9563	**197** 0	60	1·9659	**195** 0	60	1·9744	**193** 0
° ′		° ′	° ′		° ′	° ′		° ′	° ′		° ′
161 0	1·9455	60	**163** 0	1·9563	60	**165** 0	1·9659	60	**167** 0	1·9744	60
2	1·9457	58	2	1·9565	58	2	1·9661	58	2	1·9745	58
4	1·9459	56	4	1·9566	56	4	1·9662	56	4	1·9746	56
6	1·9461	54	6	1·9568	54	6	1·9664	54	6	1·9748	54
8	1·9463	52	8	1·9570	52	8	1·9665	52	8	1·9749	52
10	1·9465	50	10	1·9572	50	10	1·9667	50	10	1·9750	50
12	1·9466	48	12	1·9573	48	12	1·9668	48	12	1·9751	48
14	1·9468	46	14	1·9575	46	14	1·9670	46	14	1·9753	46
16	1·9470	44	16	1·9577	44	16	1·9671	44	16	1·9754	44
18	1·9472	42	18	1·9578	42	18	1·9673	42	18	1·9755	42
20	1·9474	40	20	1·9580	40	20	1·9674	40	20	1·9757	40
22	1·9476	38	22	1·9582	38	22	1·9676	38	22	1·9758	38
24	1·9478	36	24	1·9583	36	24	1·9677	36	24	1·9759	36
26	1·9480	34	26	1·9585	34	26	1·9679	34	26	1·9760	34
28	1·9481	32	28	1·9587	32	28	1·9680	32	28	1·9762	32
30	1·9483	30	30	1·9588	30	30	1·9681	30	30	1·9763	30
32	1·9485	28	32	1·9590	28	32	1·9683	28	32	1·9764	28
34	1·9487	26	34	1·9591	26	34	1·9684	26	34	1·9765	26
36	1·9489	24	36	1·9593	24	36	1·9686	24	36	1·9767	24
38	1·9491	22	38	1·9595	22	38	1·9687	22	38	1·9768	22
40	1·9492	20	40	1·9596	20	40	1·9689	20	40	1·9769	20
42	1·9494	18	42	1·9598	18	42	1·9690	18	42	1·9770	18
44	1·9496	16	44	1·9600	16	44	1·9692	16	44	1·9772	16
46	1·9498	14	46	1·9601	14	46	1·9693	14	46	1·9773	14
48	1·9500	12	48	1·9603	12	48	1·9694	12	48	1·9774	12
50	1·9502	10	50	1·9605	10	50	1·9696	10	50	1·9775	10
52	1·9503	8	52	1·9606	8	52	1·9697	8	52	1·9777	8
54	1·9505	6	54	1·9608	6	54	1·9699	6	54	1·9778	6
56	1·9507	4	56	1·9609	4	56	1·9700	4	56	1·9779	4
58	1·9509	2	58	1·9611	2	58	1·9702	2	58	1·9780	2
60	1·9511	**198** 0	60	1·9613	**196** 0	60	1·9703	**194** 0	60	1·9781	**192** 0

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168 0	1·9781	60	**170** 0	1·9848	60	**172** 0	1·9903	60	**174** 0	1·9945	60
2	1·9783	58	2	1·9849	58	2	1·9903	58	2	1·9946	58
4	1·9784	56	4	1·9850	56	4	1·9904	56	4	1·9946	56
6	1·9785	54	6	1·9851	54	6	1·9905	54	6	1·9947	54
8	1·9786	52	8	1·9852	52	8	1·9906	52	8	1·9948	52
10	1·9787	50	10	1·9853	50	10	1·9907	50	10	1·9948	50
12	1·9789	48	12	1·9854	48	12	1·9907	48	12	1·9949	48
14	1·9790	46	14	1·9855	46	14	1·9908	46	14	1·9949	46
16	1·9791	44	16	1·9856	44	16	1·9909	44	16	1·9950	44
18	1·9792	42	18	1·9857	42	18	1·9910	42	18	1·9951	42
20	1·9793	40	20	1·9858	40	20	1·9911	40	20	1·9951	40
22	1·9795	38	22	1·9859	38	22	1·9911	38	22	1·9952	38
24	1·9796	36	24	1·9860	36	24	1·9912	36	24	1·9952	36
26	1·9797	34	26	1·9861	34	26	1·9913	34	26	1·9953	34
28	1·9798	32	28	1·9862	32	28	1·9914	32	28	1·9953	32
30	1·9799	30	30	1·9863	30	30	1·9914	30	30	1·9954	30
32	1·9800	28	32	1·9864	28	32	1·9915	28	32	1·9955	28
34	1·9802	26	34	1·9865	26	34	1·9916	26	34	1·9955	26
36	1·9803	24	36	1·9866	24	36	1·9917	24	36	1·9956	24
38	1·9804	22	38	1·9867	22	38	1·9917	22	38	1·9956	22
40	1·9805	20	40	1·9868	20	40	1·9918	20	40	1·9957	20
42	1·9806	18	42	1·9869	18	42	1·9919	18	42	1·9957	18
44	1·9807	16	44	1·9869	16	44	1·9920	16	44	1·9958	16
46	1·9808	14	46	1·9870	14	46	1·9920	14	46	1·9958	14
48	1·9810	12	48	1·9871	12	48	1·9921	12	48	1·9959	12
50	1·9811	10	50	1·9872	10	50	1·9922	10	50	1·9959	10
52	1·9812	8	52	1·9873	8	52	1·9923	8	52	1·9960	8
54	1·9813	6	54	1·9874	6	54	1·9923	6	54	1·9960	6
56	1·9814	4	56	1·9875	4	56	1·9924	4	56	1·9961	4
58	1·9815	2	58	1·9876	2	58	1·9925	2	58	1·9961	2
60	1·9816	**191** 0	60	1·9877	**189** 0	60	1·9925	**187** 0	60	1·9962	**185** 0

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169 0	1·9816	60	**171** 0	1·9877	60	**173** 0	1·9925	60	**175** 0	1·9962	60
2	1·9817	58	2	1·9878	58	2	1·9926	58	2	1·9962	58
4	1·9818	56	4	1·9879	56	4	1·9927	56	4	1·9963	56
6	1·9820	54	6	1·9880	54	6	1·9928	54	6	1·9963	54
8	1·9821	52	8	1·9880	52	8	1·9928	52	8	1·9964	52
10	1·9822	50	10	1·9881	50	10	1·9929	50	10	1·9964	50
12	1·9823	48	12	1·9882	48	12	1·9930	48	12	1·9965	48
14	1·9824	46	14	1·9883	46	14	1·9930	46	14	1·9965	46
16	1·9825	44	16	1·9884	44	16	1·9931	44	16	1·9966	44
18	1·9826	42	18	1·9885	42	18	1·9932	42	18	1·9966	42
20	1·9827	40	20	1·9886	40	20	1·9932	40	20	1·9967	40
22	1·9828	38	22	1·9887	38	22	1·9933	38	22	1 9967	38
24	1·9829	36	24	1·9888	36	24	1·9934	36	24	1·9968	36
26	1·9830	34	26	1·9888	34	26	1·9934	34	26	1·9968	34
28	1·9831	32	28	1·9889	32	28	1·9935	32	28	1·9969	32
30	1·9833	30	30	1·9890	30	30	1·9936	30	30	1·9969	30
32	1·9834	28	32	1·9891	28	32	1·9936	28	32	1·9970	28
34	1·9835	26	34	1·9892	26	34	1·9937	26	34	1·9970	26
36	1·9836	24	36	1·9893	24	36	1·9938	24	36	1·9971	24
38	1·9837	22	38	1·9894	22	38	1·9938	22	38	1·9971	22
40	1·9838	20	40	1·9894	20	40	1·9939	20	40	1·9971	20
42	1·9839	18	42	1·9895	18	42	1·9940	18	42	1·9972	18
44	1·9840	16	44	1·9896	16	44	1·9940	16	44	1·9972	16
46	1·9841	14	46	1·9897	14	46	1·9941	14	46	1·9973	14
48	1·9842	12	48	1·9898	12	48	1·9942	12	48	1·9973	12
50	1·9843	10	50	1·9899	10	50	1·9942	10	50	1·9974	10
52	1·9844	8	52	1·9899	8	52	1·9943	8	52	1·9974	8
54	1·9845	6	54	1·9900	6	54	1·9943	6	54	1·9974	6
56	1·9846	4	56	1·9901	4	56	1·9944	4	56	1·9975	4
58	1·9847	2	58	1·9902	2	58	1·9945	2	58	1·9975	2
60	1·9848	**190** 0	60	1·9903	**188** 0	60	1·9945	**186** 0	60	1·9976	**184** 0

° ′		° ′	° ′		° ′	° ′		° ′	° ′		° ′
176 0	1·9976	60	**177** 0	1·9986	60	**178** 0	1·9994	60	**179** 0	1·9998	60
2	1·9976	58	2	1·9987	58	2	1·9994	58	2	1·9999	58
4	1·9976	56	4	1·9987	56	4	1·9994	56	4	1·9999	56
6	1·9977	54	6	1·9987	54	6	1·9995	54	6	1·9999	54
8	1·9977	52	8	1·9987	52	8	1·9995	52	8	1·9999	52
10	1·9978	50	10	1·9988	50	10	1·9995	50	10	1·9999	50
12	1·9978	48	12	1·9988	48	12	1·9995	48	12	1·9999	48
14	1·9978	46	14	1·9988	46	14	1·9995	46	14	1·9999	46
16	1·9979	44	16	1·9989	44	16	1·9995	44	16	1·9999	44
18	1·9979	42	18	1·9989	42	18	1·9996	42	18	1·9999	42
20	1·9980	40	20	1·9989	40	20	1·9996	40	20	1·9999	40
22	1·9980	38	22	1·9989	38	22	1·9996	38	22	1·9999	38
24	1·9980	36	24	1·9990	36	24	1·9996	36	24	1·9999	36
26	1·9981	34	26	1·9990	34	26	1·9996	34	26	2·0000	34
28	1·9981	32	28	1·9990	32	28	1·9996	32	28	2·0000	32
30	1·9981	30	30	1·9990	30	30	1·9997	30	30	2·0000	30
32	1·9982	28	32	1·9991	28	32	1·9997	28	32	2·0000	28
34	1·9982	26	34	1·9991	26	34	1·9997	26	34	2·0000	26
36	1·9982	24	36	1·9991	24	36	1·9997	24	36	2·0000	24
38	1·9983	22	38	1·9991	22	38	1·9997	22	38	2·0000	22
40	1·9983	20	40	1·9992	20	40	1·9997	20	40	2·0000	20
42	1·9983	18	42	1·9992	18	42	1·9997	18	42	2·0000	18
44	1·9984	16	44	1·9992	16	44	1·9998	16	44	2·0000	16
46	1·9984	14	46	1·9992	14	46	1·9998	14	46	2·0000	14
48	1·9984	12	48	1·9993	12	48	1·9998	12	48	2·0000	12
50	1·9985	10	50	1·9993	10	50	1·9998	10	50	2·0000	10
52	1·9985	8	52	1·9993	8	52	1·9998	8	52	2·0000	8
54	1·9985	6	54	1·9993	6	54	1·9998	6	54	2·0000	6
56	1·9986	4	56	1·9993	4	56	1·9998	4	56	2·0000	4
58	1·9986	2	58	1·9994	2	58	1·9998	2	58	2·0000	2
60	1·9986	**183** 0	60	1·9994	**182** 0	60	1·9998	**181** 0	60	2·0000	**180** 0

THE END.

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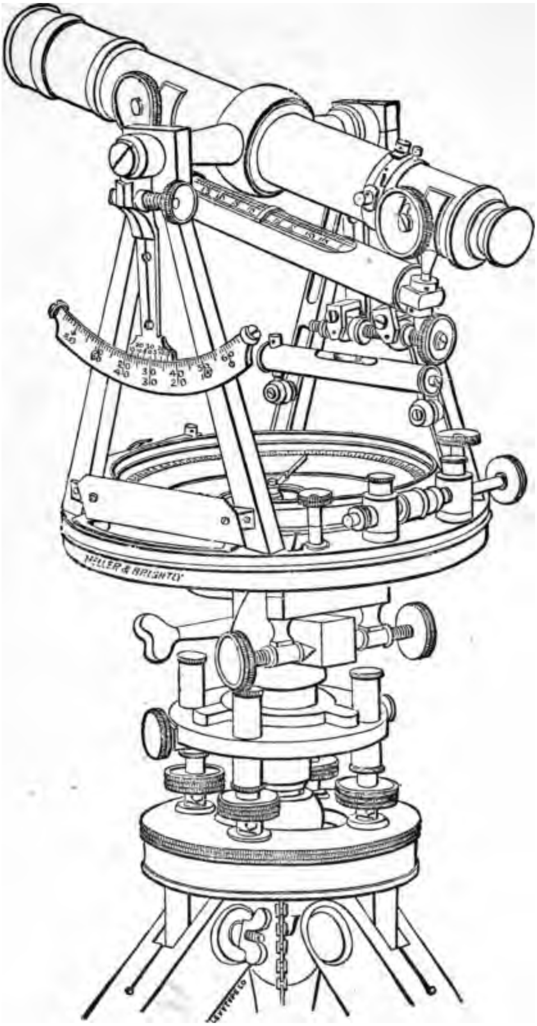

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